

# An AESOP Example: Analysis of a Misaligned Beam Compressor

Marc A. Murison

*U.S. Naval Observatory  
3450 Massachusetts Ave., NW  
Washington, DC 20392  
<mailto:murison@riemann.usno.navy.mil>  
<http://riemann.usno.navy.mil/AESOP/>*

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## ■ Trace a Ray through the System

### ■ Introduction

For this example, we consider a simple beam compressor, which consists of two confocal paraboloidal mirrors such that, when perfectly aligned, a flat input beam of radius  $R$  is compressed to a flat output beam of radius  $R/C$ , where  $C$  is the compression ratio. The driver program **BeamComp()** illustrates how to use AESOP to create an optical system and trace a ray through it. **BeamComp()** is written so that we can apply up to two independent perturbations -- translation perpendicular to the optical axis or rotation -- to either or both optical elements. We can also skew the input wavefront. In this example, we'll rotate and translate the primary mirror. First, we load the driver program:

```
> restart;
> read`d:/optics/BeamCompressor.p`;
=====
#           AESOP          #
# (An Extensible Symbolic Optics Processor) #
#           version 96.10.02      #
=====
#           Marc A. Murison    #
#           U.S. Naval Observatory #
#           Astronomical Applications Dept. #
#           murison@riemann.usno.navy.mil #
=====

Use:
BeamComp( pert_type1::integer, #first perturbation type
```

```

    eps1::name,           #name of first pert. variable
    pert_type2::integer, #second perturbation type
    eps2::name,           #name of second pert. variable
    order_eps1::integer, #expansion order of first pert.
    order_eps2::integer, #expansion order of second pert.
    order_r::integer     #radial expansion order
)

```

Perturbation Types:

TRANS_PRIMARY	translation of primary mirror in X
ROT_PRIMARY	rotation of primary mirror around Y axis
TRANS_SECONDARY	translation of secondary mirror in X
ROT_SECONDARY	rotation of secondary mirror around Y axis
FIELD	rotation of the input ray around X axis
NOTHING	do nothing

Example:

```

verbosity := 1:
assume( Delta >= 0, psi >= 0 ):
BeamComp( TRANS_PRIMARY, Delta, ROT_SECONDARY, psi, 2, 10 );

```

NOTE: you MUST assume(eps1 >= 0): and assume(eps2 >= 0):  
before calling BeamComp()

[ Next, we tell Maple that our perturbation parameters are real numbers:

[ > assume( psi >= 0, Delta >= 0 );

[ A verbosity level of 1 will give us textual information as to the progress of the ray trace.  
Levels 2 and 3 will give increasing intermediate expression verbiage.

[ > verbosity := 1:

[ Now we make the run. Let's look at simultaneous translation (along X direction) and rotation (around Y axis) of the primary mirror. We'll expand to third order in the perturbation parameters and to twelfth order in radius. The expansion to 12th order in radius means the resulting output wavefront is good to  $\frac{\rho}{f} \sim 10^{-12}$ , or the picometer (or better) regime.

## AESOP Run

```

> BeamComp( TRANS_PRIMARY, Delta, ROT_PRIMARY, psi, 3, 12 );
BeamComp_defs[]: Optical element definitions...
BeamComp_defs[]: ...primary mirror...
BeamComp_defs[]: ...secondary mirror...
BeamComp_defs[]: ...detector plane...
BeamComp_defs[1]: ...input beam...
BeamComp_defs[1]: primary mirror:

table([
  flen = f
  type = mirror
])

```

```


$$eqn = z - \frac{1}{4} \frac{x^2 + y^2}{f}$$

dir = [sin(ψ), 0, cos(ψ)]
cc = 0
pos = [Δ, 0, 0]
coord = [x, y, z]
])
BeamComp_defs[1]: secondary mirror:

table([
flen =  $\frac{f}{C}$ 
type = mirror

$$eqn = z - \frac{1}{4} \frac{(x^2 + y^2) C}{f}$$

dir = [0, 0, 1]
cc = 0
pos =  $\left[ 0, 0, \frac{f(C-1)}{C} \right]$ 
coord = [x, y, z]
])
BeamComp_defs[1]: end plane:

table([
flen = ∞
type = mirror
eqn = z
dir = [0, 0, 1]
cc = 0
pos = [0, 0, -d]
coord = [x, y, z]
])
BeamComp_defs[1]: incident beam:

table([
type = beam
dir = [0, 0, -1]
pos = [x₀, y₀, z₀]
path = 0
])
raytrace[1]: Calculating reflection 1...

```

```
reflect[1]: Transforming input beam to LOCAL frame...
reflect[1]: Finding intersection point...
beam_intersect[2]: Equation substitutions...
beam_intersect[2]: Expanding and simplifying substitution result...
beam_intersect[3]: Solving for t...
beam_intersect[4]: Calculating intersection point...
beam_intersect[4]: Trying solution 1...
intersect_findpoint[4]: Expanding in series...
intersect_findpoint[6]: Adding vector to previous position and simplifying...
intersect_findpoint[6]: Expanding in series...
beam_intersect[9]: Checking for small divisors...
beam_intersect[9]: Calculating path length...
reflect[11]: Expanding intersection point...
reflect[13]: Simplifying intersection point...
reflect[16]: Finding surface normal...
reflect[16]: Expanding normal vector...
reflect[18]: Simplifying normal vector...
reflect[20]: Calculating reflected beam...
reflect[20]: Expanding reflection vector...
reflect[31]: Simplifying reflection vector...
reflect[45]: Transforming output beam to GLOBAL frame...
reflect[45]: ...direction...
reflect[46]: ...position...
reflect[46]: Expanding position...
reflect[48]: Simplifying position...
reflect[50]: Expanding direction...
reflect[53]: Simplifying direction...
reflect[64]: Calculating optical path...
reflect[64]: Expanding optical path...
reflect[65]: Simplifying optical path...
raytrace[66]: Simplifying beam...
raytrace[80]: Calculating reflection 2...
reflect[80]: Transforming input beam to LOCAL frame...
reflect[81]: Finding intersection point...
beam_intersect[81]: Equation substitutions...
beam_intersect[81]: Expanding and simplifying substitution result...
beam_intersect[96]: Solving for t...
beam_intersect[131]: Calculating intersection point...
beam_intersect[131]: Trying solution 1...
intersect_findpoint[142]: Expanding in series...
intersect_findpoint[168]: Adding vector to previous position and simplifying...
intersect_findpoint[180]: Expanding in series...
beam_intersect[193]: Checking for small divisors...
beam_intersect[198]: WARNING! Small divisors found in this solution.
beam_intersect[198]: Trying solution 2...
intersect_findpoint[198]: Expanding in series...
intersect_findpoint[223]: Adding vector to previous position and simplifying...
intersect_findpoint[233]: Expanding in series...
beam_intersect[242]: Checking for small divisors...
```

```

beam_intersect[251]: Calculating path length...
reflect[301]: Expanding intersection point...
reflect[309]: Simplifying intersection point...
reflect[343]: Finding surface normal...
reflect[345]: Expanding normal vector...
reflect[350]: Simplifying normal vector...
reflect[368]: Calculating reflected beam...
reflect[430]: Expanding reflection vector...
reflect[778]: Simplifying reflection vector...
reflect[1076]: Transforming output beam to GLOBAL frame...
reflect[1076]: ...direction...
reflect[1077]: ...position...
reflect[1078]: Expanding position...
reflect[1082]: Simplifying position...
reflect[1112]: Expanding direction...
reflect[1116]: Simplifying direction...
reflect[1142]: Calculating optical path...
reflect[1142]: Expanding optical path...
reflect[1144]: Simplifying optical path...
raytrace[1149]: Simplifying beam...
raytrace[1207]: Calculating detector intersection point...
beam_intersect[1208]: Equation substitutions...
beam_intersect[1210]: Expanding and simplifying substitution result.
..
beam_intersect[1227]: Solving for t...
beam_intersect[1227]: Calculating intersection point...
beam_intersect[1227]: Trying solution 1...
intersect_findpoint[1229]: Expanding in series...
intersect_findpoint[1234]: Adding vector to previous position and si
mplifying...
intersect_findpoint[1257]: Expanding in series...
beam_intersect[1267]: Checking for small divisors...
beam_intersect[1276]: Transforming to GLOBAL frame...
beam_intersect[1279]: Expanding in series...
beam_intersect[1287]: Calculating path length...
raytrace[1301]: Simplifying...
raytrace[1348]: Calculating and simplifying total path length...
BeamComp[1363]: Calculating OPD...
BeamComp[1390]: Calculating aperture averaged OPD...
annular_average[1390]: Calculating average over annulus...
BeamComp[1406]: Done!

```

We're finished with the ray trace. This run took 1,406 seconds on a 90 MHz Pentium with 48 M of RAM. An insignificant amount of that time was used in virtual swap file access; hence the calculation was CPU bound. Peak memory usage, as reported on the Maple status bar, was 22.1 M. Resources required scale nonlinearly with the expansion orders used. For example, expanding to 10th order in radius instead of 12th order cuts peak memory to ~13 M. The OPD and the aperture-averaged OPD are now stored in the global variables *OPD* and *OPD\_avg*.

## The Optical Path Difference (i.e., the Wavefront)

The OPD we just calculated is:

> OPD;

$$\begin{aligned}
& \left( \left( \frac{1}{1024} \frac{C^2 (-280 + 359 C) d}{f^{14}} + \frac{1}{1024} \frac{-878 C^3 + C + 359 C^4 + 14 + 405 C^2}{f^{13} C} \right) \rho^{11} \right. \\
& + \left( -\frac{7}{128} \frac{C^2 (-10 + 11 C) d}{f^{12}} - \frac{1}{128} \frac{77 C^4 + 7 - 2 C - 189 C^3 + 90 C^2}{f^{11} C} \right) \rho^9 \\
& + \left( \frac{3}{32} \frac{C^2 (9 C - 10) d}{f^{10}} + \frac{1}{32} \frac{30 C + 3 - 64 C^2 + 27 C^3}{f^9} \right) \rho^7 \\
& + \left( -\frac{1}{8} \frac{C^2 (-10 + 7 C) d}{f^8} - \frac{1}{16} \frac{14 C^3 + 16 C - 36 C^2 + 3}{f^7} \right) \rho^5 \\
& + \left. \left( \frac{1}{2} \frac{C^2 (C - 2) d}{f^6} + \frac{1}{4} \frac{1 + 2 C^3 - 6 C^2 + 2 C}{f^5} \right) \rho^3 \right) \cos(\phi)^3 + \left( \right. \\
& \left( \frac{7}{2048} \frac{C^2 (13 C - 19) d}{f^{14}} + \frac{1}{2048} \frac{-289 C^3 - 5 C + 91 C^4 + 14 + 198 C^2}{f^{13} C} \right) \rho^{11} \\
& + \left( -\frac{5}{512} \frac{C^2 (-17 + 11 C) d}{f^{12}} - \frac{1}{1024} \frac{28 - 349 C^3 + 230 C^2 + 110 C^4 - 13 C}{f^{11} C} \right) \rho^9 \\
& + \left( \frac{5}{64} \frac{C^2 (3 C - 5) d}{f^{10}} + \frac{1}{64} \frac{15 C^3 + 3 - 47 C^2 + 28 C}{f^9} \right) \rho^7 \\
& + \left( -\frac{1}{16} \frac{C^2 (-13 + 7 C) d}{f^8} - \frac{1}{64} \frac{50 C + 9 + 28 C^3 - 90 C^2}{f^7} \right) \rho^5 \\
& + \left. \left( \frac{1}{8} \frac{C^2 (-11 + 5 C) d}{f^6} + \frac{1}{8} \frac{-17 C^2 + 3 + 8 C + 5 C^3}{f^5} \right) \rho^3 \right. \\
& + \left. \left( -\frac{1}{2} \frac{C^2 (-3 + C) d}{f^4} - \frac{1}{4} \frac{3 + 2 C - 8 C^2 + 2 C^3}{f^3} \right) \rho \right) \cos(\phi) \Delta^3 + \left( \left( \right. \right. \\
& \left( \frac{1}{256} \frac{C^2 (313 C - 165) d}{f^{13}} + \frac{1}{1024} \frac{1252 C^4 + 996 C^2 - 2754 C^3 - 12 + 7 C}{f^{12} C} \right) \rho^{11} \\
& + \left. \left. \left( -\frac{1}{128} \frac{C^2 (-180 + 281 C) d}{f^{11}} - \frac{1}{256} \frac{-12 + 488 C^2 + 10 C - 1233 C^3 + 562 C^4}{f^{10} C} \right) \rho^9 \right) \rho
\end{aligned}$$

$$\begin{aligned}
& + \left( \frac{1}{32} \frac{C^2 (-86 + 105 C) d}{f^9} + \frac{1}{32} \frac{C (-221 C + 105 C^2 + 97)}{f^8} \right) \rho^7 \\
& + \left( -\frac{3}{8} \frac{C^2 (-11 + 10 C) d}{f^7} - \frac{1}{16} \frac{1 + 60 C^3 + 62 C - 136 C^2}{f^6} \right) \rho^5 \\
& + \left( \frac{1}{2} \frac{C^2 (-8 + 5 C) d}{f^5} + \frac{1}{4} \frac{-26 C^2 + 10 C + 1 + 10 C^3}{f^4} \right) \rho^3 \right) \cos(\phi)^3 + \left( \right. \\
& \left. \frac{3}{2048} \frac{C^2 (-145 + 117 C) d}{f^{13}} + \frac{1}{4096} \frac{1302 C^2 - 29 - 2080 C^3 + 702 C^4 + 43 C}{f^{12} C} \right) \rho^{11} \\
& + \left( -\frac{5}{512} \frac{C^2 (44 C - 57) d}{f^{11}} - \frac{1}{1024} \frac{-29 - 1293 C^3 + 42 C + 440 C^4 + 780 C^2}{f^{10} C} \right) \rho^9 \\
& + \left( \frac{3}{64} \frac{C^2 (-29 + 21 C) d}{f^9} + \frac{3}{128} \frac{-121 C^2 + 42 C^3 + 67 C + 2}{f^8} \right) \rho^7 \\
& + \left( -\frac{3}{32} \frac{C^2 (-32 + 21 C) d}{f^7} - \frac{1}{64} \frac{126 C^3 + 196 C - 368 C^2 + 11}{f^6} \right) \rho^5 \\
& + \left( \frac{5}{8} \frac{C^2 (5 C - 9) d}{f^5} + \frac{1}{8} \frac{37 C + 25 C^3 - 76 C^2 + 5}{f^4} \right) \rho^3 \\
& + \left. \left( -\frac{3}{2} \frac{C^2 (2 C - 5) d}{f^3} - \frac{3}{4} \frac{4 C^3 + 4 C + 3 - 14 C^2}{f^2} \right) \rho \right) \cos(\phi) \Bigg) \psi + \left( \right. \\
& \left. \frac{7}{256} \frac{C^2 d}{f^{14}} + \frac{1}{2048} \frac{56 C^2 - 6 - 81 C}{f^{13}} \right) \rho^{12} + \left( -\frac{35}{512} \frac{C^2 d}{f^{12}} - \frac{5}{512} \frac{7 C^2 - 1 - 9 C}{f^{11}} \right) \rho^{10} \\
& + \left( \frac{5}{32} \frac{C^2 d}{f^{10}} + \frac{1}{32} \frac{5 C^2 - 5 C - 1}{f^9} \right) \rho^8 + \left( -\frac{5}{16} \frac{C^2 d}{f^8} - \frac{1}{32} \frac{-3 + 10 C^2 - 8 C}{f^7} \right) \rho^6 \\
& + \left( \frac{1}{2} \frac{C^2 d}{f^6} + \frac{1}{4} \frac{(2 C + 1)(C - 1)}{f^5} \right) \rho^4 + \left( -\frac{1}{2} \frac{C^2 d}{f^4} - \frac{1}{2} \frac{(C - 1)(C + 1)}{f^3} \right) \rho^2 \Bigg) \cos(\phi)^2 \\
& + \left( \frac{7}{8192} \frac{C^2 d}{f^{14}} + \frac{1}{8192} \frac{-2 + 7 C^2 - 12 C}{f^{13}} \right) \rho^{12} + \left( -\frac{3}{1024} \frac{C^2 d}{f^{12}} - \frac{1}{1024} \frac{-5 C - 1 + 3 C^2}{f^{11}} \right) \rho^{10} \\
& + \left( \frac{5}{512} \frac{C^2 d}{f^{10}} + \frac{1}{512} \frac{-2 + 5 C^2 - 8 C}{f^9} \right) \rho^8 + \left( -\frac{1}{32} \frac{C^2 d}{f^8} - \frac{1}{64} \frac{-1 + 2 C^2 - 3 C}{f^7} \right) \rho^6
\end{aligned}$$

$$\begin{aligned}
& + \left( \frac{3}{32} \frac{C^2 d}{f^6} + \frac{1}{32} \frac{3 C^2 - 4 C - 2}{f^5} \right) p^4 + \left( -\frac{1}{4} \frac{C^2 d}{f^4} - \frac{1}{4} \frac{-1 + C^2 - C}{f^3} \right) p^2 + \frac{1}{2} \frac{C^2 d}{f^2} + \frac{1}{2} \frac{-2 + C^2}{f} \\
& \left. \right) \Delta^2 + \left( \left( \left( \frac{3}{1024} \frac{C^2 (-150 + 479 C) d}{f^{12}} + \frac{1}{1024} \frac{-2862 C^3 + 2 C - 21 + 731 C^2 + 1437 C^4}{f^{11} C} \right) p^{11} \right. \right. \\
& + \left. \left. - \frac{1}{128} \frac{C^2 (-142 + 335 C) d}{f^{10}} - \frac{1}{256} \frac{414 C^2 + 670 C^4 + 8 C - 21 - 1330 C^3}{f^9 C} \right) p^9 \right. \\
& + \left. \left. \frac{1}{32} \frac{C^2 (-79 + 132 C) d}{f^8} + \frac{1}{32} \frac{(2 C - 1)(66 C^2 - 93 C + 4)}{f^7} \right) p^7 \right. \\
& + \left. \left. - \frac{1}{8} \frac{C^2 (-36 + 41 C) d}{f^6} - \frac{1}{8} \frac{-85 C^2 + 41 C^3 + 39 C - 2}{f^5} \right) p^5 \right. \\
& + \left. \left. \left. \frac{1}{2} \frac{C^2 (8 C - 11) d}{f^4} + \frac{1}{4} \frac{-38 C^2 - 1 + 16 C + 16 C^3}{f^3} \right) p^3 \right) \cos(\phi)^3 + \left( \right. \right. \\
& \left. \left. \frac{1}{2048} \frac{C^2 (-375 + 442 C) d}{f^{12}} + \frac{1}{2048} \frac{-1141 C^3 - 9 C - 1 + 442 C^4 + 568 C^2}{f^{11} C} \right) p^{11} \right. \\
& + \left. \left. \left. - \frac{1}{512} \frac{C^2 (-257 + 286 C) d}{f^{10}} - \frac{1}{1024} \frac{-2 + 720 C^2 - 13 C - 1463 C^3 + 572 C^4}{f^9 C} \right) p^9 \right. \\
& + \left. \left. \left. \frac{1}{128} \frac{C^2 (171 C - 167) d}{f^8} + \frac{1}{128} \frac{-5 - 429 C^2 + 171 C^3 + 204 C}{f^7} \right) p^7 \right. \\
& + \left. \left. \left. - \frac{1}{32} \frac{C^2 (91 C - 101) d}{f^6} - \frac{1}{64} \frac{-5 - 464 C^2 + 226 C + 182 C^3}{f^5} \right) p^5 \right. \\
& + \left. \left. \left. \frac{5}{8} \frac{C^2 (8 C - 11) d}{f^4} + \frac{1}{8} \frac{C (40 C - 27) (C - 2)}{f^3} \right) p^3 \right. \\
& + \left. \left. \left. - \frac{1}{2} \frac{C^2 (12 C - 25) d}{f^2} - \frac{1}{2} \frac{12 C^3 + 4 + 12 C - 37 C^2}{f} \right) p \right) \cos(\phi) \right) \Psi^2 + \left( \right. \\
& \left. \left. \left. \frac{59}{1024} \frac{C^2 d}{f^{13}} + \frac{1}{1024} \frac{-86 C - 1 + 59 C^2}{f^{12}} \right) p^{12} + \left( -\frac{75}{512} \frac{C^2 d}{f^{11}} - \frac{1}{512} \frac{75 C^2 - 98 C - 2}{f^{10}} \right) p^{10} \right. \\
& + \left. \left. \left. \frac{11}{32} \frac{C^2 d}{f^9} + \frac{1}{64} \frac{-1 - 23 C + 22 C^2}{f^8} \right) p^8 + \left( -\frac{23}{32} \frac{C^2 d}{f^7} - \frac{1}{32} \frac{23 C^2 - 20 C - 2}{f^6} \right) p^6 \right)
\end{aligned}$$

$$\begin{aligned}
& + \left( \frac{5}{4} \frac{C^2 d}{f^5} + \frac{1}{4} \frac{5 C^2 - 1 - 3 C}{f^4} \right) p^4 + \left( -\frac{3}{2} \frac{C^2 d}{f^3} - \frac{1}{2} \frac{3 C^2 - 2}{f^2} \right) p^2 \right) \cos(\phi)^2 \\
& + \left( \frac{1}{512} \frac{C^2 d}{f^{13}} + \frac{1}{2048} \frac{4 C^2 - 1 - 7 C}{f^{12}} \right) p^{12} + \left( -\frac{7}{1024} \frac{C^2 d}{f^{11}} - \frac{1}{1024} \frac{-2 + 7 C^2 - 12 C}{f^{10}} \right) p^{10} \\
& + \left( \frac{3}{128} \frac{C^2 d}{f^9} + \frac{1}{128} \frac{-5 C - 1 + 3 C^2}{f^8} \right) p^8 + \left( -\frac{5}{64} \frac{C^2 d}{f^7} - \frac{1}{64} \frac{-2 + 5 C^2 - 8 C}{f^6} \right) p^6 \\
& + \left( \frac{1}{4} \frac{C^2 d}{f^5} + \frac{1}{8} \frac{-1 + 2 C^2 - 3 C}{f^4} \right) p^4 + \left( -\frac{3}{4} \frac{C^2 d}{f^3} - \frac{1}{4} \frac{3 C^2 - 4 C - 2}{f^2} \right) p^2 + 2 \frac{C^2 d}{f} - 4 + 2 C^2 \\
& \left. \right) \Psi + \left( -\frac{1}{1024} \frac{p^{11}}{f^{11}} + \frac{1}{256} \frac{p^9}{f^9} - \frac{1}{64} \frac{p^7}{f^7} + \frac{1}{16} \frac{p^5}{f^5} - \frac{1}{4} \frac{p^3}{f^3} + \frac{p}{f} \right) \cos(\phi) \right) \Delta + \left( \right. \\
& \left( \frac{1}{512} \frac{C^2 (272 C - 35) d}{f^{11}} + \frac{1}{1024} \frac{544 C^3 + 140 C - 986 C^2 + 1}{f^{10}} \right) p^{11} \\
& + \left( -\frac{1}{128} \frac{C^2 (131 C - 32) d}{f^9} - \frac{1}{256} \frac{106 C - 475 C^2 + 262 C^3 - 1}{f^8} \right) p^9 \\
& + \left( \frac{1}{32} \frac{C^2 (54 C - 23) d}{f^7} + \frac{1}{32} \frac{-95 C^2 + 34 C + 54 C^3 - 1}{f^6} \right) p^7 \\
& + \left( -\frac{1}{8} \frac{C^2 (18 C - 13) d}{f^5} - \frac{1}{8} \frac{-1 + 16 C - 35 C^2 + 18 C^3}{f^4} \right) p^5 \\
& + \left. \left( \frac{1}{2} \frac{C^2 (-5 + 4 C) d}{f^3} + \frac{1}{4} \frac{(4 C - 1)(2 C^2 - 4 C + 1)}{f^2} \right) p^3 \right) \cos(\phi)^3 + \left( \right. \\
& \left( \frac{1}{2048} \frac{C^2 (-67 + 182 C) d}{f^{11}} + \frac{1}{12288} \frac{-9 C + 17 + 630 C^2 - 2304 C^3 + 1092 C^4}{f^{10} C} \right) p^{11} \\
& + \left( -\frac{1}{512} \frac{C^2 (121 C - 52) d}{f^9} - \frac{1}{3072} \frac{17 - 1527 C^3 - 12 C + 726 C^4 + 462 C^2}{f^8 C} \right) p^9 \\
& + \left( \frac{3}{128} \frac{C^2 (25 C - 13) d}{f^7} + \frac{1}{384} \frac{225 C^3 + 159 C + 1 - 468 C^2}{f^6} \right) p^7 \\
& + \left. \left( -\frac{7}{16} \frac{C^2 (-2 + 3 C) d}{f^5} - \frac{1}{192} \frac{216 C + 252 C^3 - 540 C^2 - 1}{f^4} \right) p^5 \right)
\end{aligned}$$

$$\begin{aligned}
& + \left( \frac{1}{8} \frac{C^2 (20C - 19)d}{f^3} + \frac{1}{24} \frac{-141C^2 + 60C^3 + 72C + 1}{f^2} \right) \rho^3 \\
& + \left( - \frac{C^2 (-7 + 4C)d}{f} + 11C^2 - 4C - 4C^3 - \frac{1}{3} \right) \rho \right) \cos(\phi) \Bigg) \psi^3 + \left( \left( \frac{31}{1024} \frac{C^2 d}{f^{12}} + \frac{1}{2048} \frac{62C^2 + 4 - 91C}{f^{11}} \right) \rho^{12} + \left( - \frac{5}{64} \frac{C^2 d}{f^{10}} - \frac{1}{512} \frac{40C^2 + 3 - 53C}{f^9} \right) \rho^{10} \right. \\
& + \left( \frac{3}{16} \frac{C^2 d}{f^8} + \frac{1}{64} \frac{(12C - 1)(C - 1)}{f^7} \right) \rho^8 + \left( - \frac{13}{32} \frac{C^2 d}{f^6} - \frac{1}{32} \frac{1 + 13C^2 - 12C}{f^5} \right) \rho^6 \\
& + \left( \frac{3}{4} \frac{C^2 d}{f^4} + \frac{1}{4} \frac{C(-2 + 3C)}{f^3} \right) \rho^4 + \left( - \frac{C^2 d}{f^2} - \frac{1}{2} \frac{-1 + 2C^2}{f} \right) \rho^2 \Big) \cos(\phi)^2 \\
& + \left( \frac{9}{8192} \frac{C^2 d}{f^{12}} + \frac{1}{8192} \frac{C(9C - 16)}{f^{11}} \right) \rho^{12} + \left( - \frac{1}{256} \frac{C^2 d}{f^{10}} - \frac{1}{1024} \frac{C(-7 + 4C)}{f^9} \right) \rho^{10} \\
& + \left( \frac{7}{512} \frac{C^2 d}{f^8} + \frac{1}{512} \frac{C(-12 + 7C)}{f^7} \right) \rho^8 + \left( - \frac{3}{64} \frac{C^2 d}{f^6} - \frac{1}{64} \frac{C(3C - 5)}{f^5} \right) \rho^6 \\
& + \left( \frac{5}{32} \frac{C^2 d}{f^4} + \frac{1}{32} \frac{C(-8 + 5C)}{f^3} \right) \rho^4 + \left( - \frac{1}{2} \frac{C^2 d}{f^2} - \frac{1}{4} \frac{2C^2 - 4C + 1}{f} \right) \rho^2 + 2C^2 d \\
& \left. + 2f(C - 1)(C + 1) \right) \psi^2 + \left( - \frac{1}{1024} \frac{\rho^{11}}{f^{10}} + \frac{1}{256} \frac{\rho^9}{f^8} - \frac{1}{64} \frac{\rho^7}{f^6} + \frac{1}{16} \frac{\rho^5}{f^4} - \frac{1}{4} \frac{\rho^3}{f^2} + 2\rho \right) \cos(\phi) \psi
\end{aligned}$$

> cost(OPD);

433 additions + 2565 multiplications + 187 divisions + 13 functions

## ■ The Aperture-Averaged OPD

[ The OPD averaged over an annular aperture of inner and outer radii  $a$  and  $b$  is:

> OPD\_avg;

$$\begin{aligned}
& \left( \left( \frac{1}{2} \frac{C^2}{f^2} - \frac{1}{4} \frac{C^2(a^2 + b^2)}{f^4} + \frac{11}{96} \frac{C^2(b^2 - ab + a^2)(ab + b^2 + a^2)}{f^6} \right. \right. \\
& \left. \left. - \frac{3}{64} \frac{C^2(a^2 + b^2)(b^4 + a^4)}{f^8} + \frac{9}{512} \frac{C^2 \% 4 \% 3}{f^{10}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. - \frac{19}{3072} \frac{C^2 (a^2 + b^2) (b^4 - a^2 b^2 + a^4) (b^2 - a b + a^2) (a b + b^2 + a^2)}{f^{12}} + \frac{17}{8192} \frac{C^2 \%2 \%1}{f^{14}} \right) \\
& d + \frac{\frac{1}{2} C^2 - 1}{f} - \frac{1}{8} \frac{(a^2 + b^2) (-2 + 2 C^2 - C)}{f^3} \\
& + \frac{1}{96} \frac{(a b + b^2 + a^2) (b^2 - a b + a^2) (11 C^2 - 8 C - 6)}{f^5} \\
& - \frac{1}{256} \frac{(a^2 + b^2) (b^4 + a^4) (-4 - 11 C + 12 C^2)}{f^7} + \frac{1}{2560} \frac{\%4 \%3 (45 C^2 - 48 C - 10)}{f^9} \\
& - \frac{1}{3072} \frac{(a^2 + b^2) (a b + b^2 + a^2) (b^2 - a b + a^2) (b^4 - a^2 b^2 + a^4) (-3 - 25 C + 19 C^2)}{f^{11}} \\
& + \frac{1}{57344} \frac{\%2 \%1 (119 C^2 - 14 - 174 C)}{f^{13}} \right) \Delta^2 + \left( \left( 2 \frac{C^2}{f} - \frac{3}{4} \frac{C^2 (a^2 + b^2)}{f^3} \right. \right. \\
& + \frac{7}{24} \frac{C^2 (a b + b^2 + a^2) (b^2 - a b + a^2)}{f^5} - \frac{7}{64} \frac{C^2 (a^2 + b^2) (b^4 + a^4)}{f^7} + \frac{5}{128} \frac{C^2 \%4 \%3}{f^9} \\
& \left. \left. - \frac{41}{3072} \frac{C^2 (a^2 + b^2) (a b + b^2 + a^2) (b^2 - a b + a^2) (b^4 - a^2 b^2 + a^4)}{f^{11}} + \frac{9}{2048} \frac{C^2 \%2 \%1}{f^{13}} \right) \right. \\
& d + 2 C^2 - 4 - \frac{1}{4} \frac{(a^2 + b^2) (-2 + 3 C^2 - 2 C)}{f^2} \\
& + \frac{1}{24} \frac{(a b + b^2 + a^2) (b^2 - a b + a^2) (7 C^2 - 6 C - 2)}{f^4} \\
& - \frac{1}{64} \frac{(a^2 + b^2) (b^4 + a^4) (-1 - 7 C + 7 C^2)}{f^6} + \frac{1}{640} \frac{\%4 \%3 (25 C^2 - 28 C - 2)}{f^8} \\
& - \frac{1}{3072} \frac{(a^2 + b^2) (a b + b^2 + a^2) (b^2 - a b + a^2) (b^4 - a^2 b^2 + a^4) (-2 - 55 C + 41 C^2)}{f^{10}} \\
& + \frac{1}{14336} \frac{\%2 \%1 (63 C^2 - 2 - 93 C)}{f^{12}} \right) \Psi \Delta + \left( \left( 2 C^2 - \frac{1}{2} \frac{C^2 (a^2 + b^2)}{f^2} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + \frac{17}{96} \frac{C^2 (a b + b^2 + a^2) (b^2 - a b + a^2)}{f^4} - \frac{1}{16} \frac{C^2 (a^2 + b^2) (b^4 + a^4)}{f^6} + \frac{11}{512} \frac{C^2 \%4 \%3}{f^8} \\
& - \frac{11}{1536} \frac{C^2 (a^2 + b^2) (a b + b^2 + a^2) (b^2 - a b + a^2) (b^4 - a^2 b^2 + a^4)}{f^{10}} + \frac{19}{8192} \frac{C^2 \%2 \%1}{f^{12}} \Big) \\
& d + 2 f (C - 1) (C + 1) - \frac{1}{2} \frac{C (a^2 + b^2) (C - 1)}{f} \\
& + \frac{1}{96} \frac{C (a b + b^2 + a^2) (b^2 - a b + a^2) (-16 + 17 C)}{f^3} \\
& - \frac{1}{256} \frac{(a^2 + b^2) (b^4 + a^4) (16 C - 1) (C - 1)}{f^5} + \frac{1}{2560} \frac{\%4 \%3 (-64 C + 4 + 55 C^2)}{f^7} \\
& - \frac{1}{6144} \frac{(a^2 + b^2) (a b + b^2 + a^2) (b^2 - a b + a^2) (b^4 - a^2 b^2 + a^4) (44 C^2 + 3 - 60 C)}{f^9} \\
& + \frac{1}{57344} \frac{\%2 \%1 (133 C^2 - 198 C + 8)}{f^{11}} \Big) \Psi^2
\end{aligned}$$

$\%1 := b^6 - b^5 a + a^2 b^4 - b^3 a^3 + a^4 b^2 - b a^5 + a^6$   
 $\%2 := a^6 + b^5 a + b^6 + a^4 b^2 + b a^5 + a^2 b^4 + b^3 a^3$   
 $\%3 := b^4 - b^3 a + a^2 b^2 - b a^3 + a^4$   
 $\%4 := b^4 + a^2 b^2 + b a^3 + a^4 + b^3 a$

> cost(OPD\_avg);

283 additions + 1236 multiplications + 39 divisions

Notice that the aperture-averaged OPD is second order in the perturbations. Also, note the cross term. Hence, we should be able to find values of  $\Delta$  and  $\Psi$  that minimize the averaged OPD. We can get a feel for the sensitivity of the averaged OPD by considering a numerical example:

> evalf( subs( a=5, b=10, f=100, d=20, C=10, OPD\_avg ) );

$$.5864543026 \Delta^2 + 234.9479888 \Psi \Delta + 23731.50613 \Psi^2$$

With OPD and  $\Delta$  expressed in microns, and  $\Psi$  in arc seconds, this is

> subs( Delta=10^(-4)\*Delta, psi=evalf(arcsec)\*psi, "\*10^4 );

$$.00005864543026 \Delta^2 + .001139059994 \Psi \Delta + .005577955379 \Psi^2$$

## Zernike Series Expansion of the Wavefront

In this section we decompose the OPD into a Zernike series. First, read the Zernike routines, then set the compression factor to 10, the distance to the detector to 20 cm, and normalize the radius (as required by Zernike expansion routine). Now  $R$  will be the input beam radius.

```
[> read`d:/optics/AESOP/zseries.p`:
```

The OPD then looks like this:

## **OPD for Zernike Series Input**

$$\begin{aligned}
 > \text{opd} := \text{collect}(\text{subs}(\text{rho}=R*\text{rho}, \text{OPD}), \\
 & [\Delta, \psi, \cos(\phi), \text{rho}, d], \text{simplify}); \\
 \text{opd} := & \left( \left( \frac{1}{1024} \frac{C^2 (-280 + 359 C) R^{11} d}{f^{14}} + \frac{1}{1024} \frac{(-878 C^3 + C + 359 C^4 + 14 + 405 C^2) R^{11}}{f^{13} C} \right) \rho^{11} \right. \\
 & + \left( -\frac{7}{128} \frac{C^2 (-10 + 11 C) R^9 d}{f^{12}} - \frac{1}{128} \frac{(77 C^4 + 7 - 2 C - 189 C^3 + 90 C^2) R^9}{f^{11} C} \right) \rho^9 \\
 & + \left( \frac{3}{32} \frac{C^2 (9 C - 10) R^7 d}{f^{10}} + \frac{1}{32} \frac{(30 C + 3 - 64 C^2 + 27 C^3) R^7}{f^9} \right) \rho^7 \\
 & + \left( -\frac{1}{8} \frac{C^2 (-10 + 7 C) R^5 d}{f^8} - \frac{1}{16} \frac{(14 C^3 + 16 C - 36 C^2 + 3) R^5}{f^7} \right) \rho^5 \\
 & + \left. \left( \frac{1}{2} \frac{C^2 (C - 2) R^3 d}{f^6} + \frac{1}{4} \frac{(1 + 2 C^3 - 6 C^2 + 2 C) R^3}{f^5} \right) \rho^3 \right) \cos(\phi)^3 + \\
 & \left( \frac{7}{2048} \frac{C^2 (13 C - 19) R^{11} d}{f^{14}} + \frac{1}{2048} \frac{(-289 C^3 - 5 C + 91 C^4 + 14 + 198 C^2) R^{11}}{f^{13} C} \right) \rho^{11} \\
 & + \left( -\frac{5}{512} \frac{C^2 (-17 + 11 C) R^9 d}{f^{12}} - \frac{1}{1024} \frac{(28 - 349 C^3 + 230 C^2 + 110 C^4 - 13 C) R^9}{f^{11} C} \right) \rho^9 \\
 & + \left( \frac{5}{64} \frac{C^2 (3 C - 5) R^7 d}{f^{10}} + \frac{1}{64} \frac{(15 C^3 + 3 - 47 C^2 + 28 C) R^7}{f^9} \right) \rho^7 \\
 & + \left( -\frac{1}{16} \frac{C^2 (-13 + 7 C) R^5 d}{f^8} - \frac{1}{64} \frac{(50 C + 9 + 28 C^3 - 90 C^2) R^5}{f^7} \right) \rho^5 \\
 & + \left( \frac{1}{8} \frac{C^2 (-11 + 5 C) R^3 d}{f^6} + \frac{1}{8} \frac{(-17 C^2 + 3 + 8 C + 5 C^3) R^3}{f^5} \right) \rho^3
 \end{aligned}$$

$$\begin{aligned}
& + \left( -\frac{1}{2} \frac{C^2 (-3+C) R d}{f^4} - \frac{1}{4} \frac{(3+2 C-8 C^2+2 C^3) R}{f^3} \right) \rho \Bigg) \cos(\phi) \Bigg) \Delta^3 + \left( \left( \right. \right. \\
& \left. \left. + \left( \frac{1}{256} \frac{C^2 (313 C-165) R^{11} d}{f^{13}} + \frac{1}{1024} \frac{(1252 C^4+996 C^2-2754 C^3-12+7 C) R^{11}}{f^{12} C} \right) \rho^{11} \right. \\
& + \left( -\frac{1}{128} \frac{C^2 (-180+281 C) R^9 d}{f^{11}} - \frac{1}{256} \frac{(-12+488 C^2+10 C-1233 C^3+562 C^4) R^9}{f^{10} C} \right) \rho^9 \\
& + \left( \frac{1}{32} \frac{C^2 (-86+105 C) R^7 d}{f^9} + \frac{1}{32} \frac{C (-221 C+105 C^2+97) R^7}{f^8} \right) \rho^7 \\
& + \left( -\frac{3}{8} \frac{C^2 (-11+10 C) R^5 d}{f^7} - \frac{1}{16} \frac{(1+60 C^3+62 C-136 C^2) R^5}{f^6} \right) \rho^5 \\
& + \left. \left. + \left( \frac{1}{2} \frac{C^2 (-8+5 C) R^3 d}{f^5} + \frac{1}{4} \frac{(-26 C^2+10 C+1+10 C^3) R^3}{f^4} \right) \rho^3 \right) \cos(\phi)^3 + \left( \right. \right. \\
& \left. \left. + \left( \frac{3}{2048} \frac{C^2 (-145+117 C) R^{11} d}{f^{13}} + \frac{1}{4096} \frac{(1302 C^2-29-2080 C^3+702 C^4+43 C) R^{11}}{f^{12} C} \right) \rho^{11} \right. \\
& + \left( -\frac{5}{512} \frac{C^2 (44 C-57) R^9 d}{f^{11}} - \frac{1}{1024} \frac{(-29-1293 C^3+42 C+440 C^4+780 C^2) R^9}{f^{10} C} \right) \rho^9 \\
& + \left( \frac{3}{64} \frac{C^2 (-29+21 C) R^7 d}{f^9} + \frac{3}{128} \frac{(-121 C^2+42 C^3+67 C+2) R^7}{f^8} \right) \rho^7 \\
& + \left( -\frac{3}{32} \frac{C^2 (-32+21 C) R^5 d}{f^7} - \frac{1}{64} \frac{(126 C^3+196 C-368 C^2+11) R^5}{f^6} \right) \rho^5 \\
& + \left( \frac{5}{8} \frac{C^2 (5 C-9) R^3 d}{f^5} + \frac{1}{8} \frac{(37 C+25 C^3-76 C^2+5) R^3}{f^4} \right) \rho^3 \\
& + \left. \left. + \left( -\frac{3}{2} \frac{C^2 (2 C-5) R d}{f^3} - \frac{3}{4} \frac{(4 C^3+4 C+3-14 C^2) R}{f^2} \right) \rho \right) \cos(\phi) \right) \psi + \left( \right. \\
& \left. \left. + \left( \frac{7}{256} \frac{C^2 R^{12} d}{f^{14}} + \frac{1}{2048} \frac{(56 C^2-6-81 C) R^{12}}{f^{13}} \right) \rho^{12} \right)
\end{aligned}$$

$$\begin{aligned}
& + \left( -\frac{35}{512} \frac{C^2 R^{10} d}{f^{12}} - \frac{5}{512} \frac{(7 C^2 - 1 - 9 C) R^{10}}{f^{11}} \right) \rho^{10} \\
& + \left( \frac{5}{32} \frac{C^2 R^8 d}{f^{10}} + \frac{1}{32} \frac{(5 C^2 - 5 C - 1) R^8}{f^9} \right) \rho^8 \\
& + \left( -\frac{5}{16} \frac{C^2 R^6 d}{f^8} - \frac{1}{32} \frac{(-3 + 10 C^2 - 8 C) R^6}{f^7} \right) \rho^6 \\
& + \left( \frac{1}{2} \frac{C^2 R^4 d}{f^6} + \frac{1}{4} \frac{(2 C + 1) (C - 1) R^4}{f^5} \right) \rho^4 + \left( -\frac{1}{2} \frac{C^2 R^2 d}{f^4} - \frac{1}{2} \frac{(C - 1) (C + 1) R^2}{f^3} \right) \rho^2 \Bigg) \\
& \cos(\phi)^2 + \left( \frac{7}{8192} \frac{C^2 R^{12} d}{f^{14}} + \frac{1}{8192} \frac{(-2 + 7 C^2 - 12 C) R^{12}}{f^{13}} \right) \rho^{12} \\
& + \left( -\frac{3}{1024} \frac{C^2 R^{10} d}{f^{12}} - \frac{1}{1024} \frac{(-5 C - 1 + 3 C^2) R^{10}}{f^{11}} \right) \rho^{10} \\
& + \left( \frac{5}{512} \frac{C^2 R^8 d}{f^{10}} + \frac{1}{512} \frac{(-2 + 5 C^2 - 8 C) R^8}{f^9} \right) \rho^8 \\
& + \left( -\frac{1}{32} \frac{C^2 R^6 d}{f^8} - \frac{1}{64} \frac{(-1 + 2 C^2 - 3 C) R^6}{f^7} \right) \rho^6 \\
& + \left( \frac{3}{32} \frac{C^2 R^4 d}{f^6} + \frac{1}{32} \frac{(3 C^2 - 4 C - 2) R^4}{f^5} \right) \rho^4 + \left( -\frac{1}{4} \frac{C^2 R^2 d}{f^4} - \frac{1}{4} \frac{(-1 + C^2 - C) R^2}{f^3} \right) \rho^2 \\
& + \frac{1}{2} \frac{C^2 d}{f^2} + \frac{1}{2} \frac{-2 + C^2}{f} \Bigg) \Delta^2 + \left( \left( \left( \left( \frac{3}{1024} \frac{C^2 (-150 + 479 C) R^{11} d}{f^{12}} + \frac{1}{1024} \frac{(-2862 C^3 + 2 C - 21 + 731 C^2 + 1437 C^4) R^{11}}{f^{11} C} \right) \rho^{11} \right. \right. \right. \right. \\
& \left. \left. \left. \left. + \left( -\frac{1}{128} \frac{C^2 (-142 + 335 C) R^9 d}{f^{10}} - \frac{1}{256} \frac{(414 C^2 + 670 C^4 + 8 C - 21 - 1330 C^3) R^9}{f^9 C} \right) \rho^9 \right. \right. \right. \right. \\
& \left. \left. \left. \left. + \left( \frac{1}{32} \frac{C^2 (-79 + 132 C) R^7 d}{f^8} + \frac{1}{32} \frac{(2 C - 1) (66 C^2 - 93 C + 4) R^7}{f^7} \right) \rho^7 \right. \right. \right. \right)
\end{aligned}$$

$$\begin{aligned}
& + \left( -\frac{1}{8} \frac{C^2 (-36 + 41C) R^5 d}{f^6} - \frac{1}{8} \frac{(-85 C^2 + 41 C^3 + 39 C - 2) R^5}{f^5} \right) \rho^5 \\
& + \left( \frac{1}{2} \frac{C^2 (8C - 11) R^3 d}{f^4} + \frac{1}{4} \frac{(-38 C^2 - 1 + 16 C + 16 C^3) R^3}{f^3} \right) \rho^3 \Bigg) \cos(\phi)^3 + \Bigg( \\
& \left( \frac{1}{2048} \frac{C^2 (-375 + 442 C) R^{11} d}{f^{12}} + \frac{1}{2048} \frac{(-1141 C^3 - 9 C - 1 + 442 C^4 + 568 C^2) R^{11}}{f^{11} C} \right) \rho^{11} \\
& + \left( -\frac{1}{512} \frac{C^2 (-257 + 286 C) R^9 d}{f^{10}} - \frac{1}{1024} \frac{(-2 + 720 C^2 - 13 C - 1463 C^3 + 572 C^4) R^9}{f^9 C} \right) \rho^9 \\
& + \left( \frac{1}{128} \frac{C^2 (171 C - 167) R^7 d}{f^8} + \frac{1}{128} \frac{(-5 - 429 C^2 + 171 C^3 + 204 C) R^7}{f^7} \right) \rho^7 \\
& + \left( -\frac{1}{32} \frac{C^2 (91 C - 101) R^5 d}{f^6} - \frac{1}{64} \frac{(-5 - 464 C^2 + 226 C + 182 C^3) R^5}{f^5} \right) \rho^5 \\
& + \left( \frac{5}{8} \frac{C^2 (8C - 11) R^3 d}{f^4} + \frac{1}{8} \frac{C (40 C - 27) (C - 2) R^3}{f^3} \right) \rho^3 \\
& + \left( -\frac{1}{2} \frac{C^2 (12 C - 25) R d}{f^2} - \frac{1}{2} \frac{(12 C^3 + 4 + 12 C - 37 C^2) R}{f} \right) \rho \Bigg) \cos(\phi) \Bigg) \psi^2 + \Bigg( \\
& \left( \frac{59}{1024} \frac{R^{12} C^2 d}{f^{13}} + \frac{1}{1024} \frac{(-86 C - 1 + 59 C^2) R^{12}}{f^{12}} \right) \rho^{12} \\
& + \left( -\frac{75}{512} \frac{R^{10} C^2 d}{f^{11}} - \frac{1}{512} \frac{(75 C^2 - 98 C - 2) R^{10}}{f^{10}} \right) \rho^{10} \\
& + \left( \frac{11}{32} \frac{R^8 C^2 d}{f^9} + \frac{1}{64} \frac{(-1 - 23 C + 22 C^2) R^8}{f^8} \right) \rho^8 \\
& + \left( -\frac{23}{32} \frac{R^6 C^2 d}{f^7} - \frac{1}{32} \frac{(23 C^2 - 20 C - 2) R^6}{f^6} \right) \rho^6 \\
& + \left( \frac{5}{4} \frac{R^4 C^2 d}{f^5} + \frac{1}{4} \frac{(5 C^2 - 1 - 3 C) R^4}{f^4} \right) \rho^4 + \left( -\frac{3}{2} \frac{R^2 C^2 d}{f^3} - \frac{1}{2} \frac{(3 C^2 - 2) R^2}{f^2} \right) \rho^2 \Bigg) \cos(\phi)^2 \\
& + \left( \frac{1}{512} \frac{R^{12} C^2 d}{f^{13}} + \frac{1}{2048} \frac{(4 C^2 - 1 - 7 C) R^{12}}{f^{12}} \right) \rho^{12}
\end{aligned}$$

$$\begin{aligned}
& + \left( -\frac{7}{1024} \frac{R^{10} C^2 d}{f^{11}} - \frac{1}{1024} \frac{(-2+7 C^2 - 12 C) R^{10}}{f^{10}} \right) \rho^{10} \\
& + \left( \frac{3}{128} \frac{R^8 C^2 d}{f^9} + \frac{1}{128} \frac{(-5 C - 1 + 3 C^2) R^8}{f^8} \right) \rho^8 \\
& + \left( -\frac{5}{64} \frac{R^6 C^2 d}{f^7} - \frac{1}{64} \frac{(-2+5 C^2 - 8 C) R^6}{f^6} \right) \rho^6 \\
& + \left( \frac{1}{4} \frac{R^4 C^2 d}{f^5} + \frac{1}{8} \frac{(-1+2 C^2 - 3 C) R^4}{f^4} \right) \rho^4 + \left( -\frac{3}{4} \frac{R^2 C^2 d}{f^3} - \frac{1}{4} \frac{(3 C^2 - 4 C - 2) R^2}{f^2} \right) \rho^2 \\
& - 4 + 2 \frac{C^2 d}{f} + 2 C^2 \Bigg) \psi \\
& + \left( -\frac{1}{1024} \frac{R^{11} \rho^{11}}{f^{11}} + \frac{1}{256} \frac{R^9 \rho^9}{f^9} - \frac{1}{64} \frac{R^7 \rho^7}{f^7} + \frac{1}{16} \frac{R^5 \rho^5}{f^5} - \frac{1}{4} \frac{R^3 \rho^3}{f^3} + \frac{R \rho}{f} \right) \cos(\phi) \Bigg) \Delta + \left( \left( \frac{1}{512} \frac{C^2 (272 C - 35) R^{11} d}{f^{11}} + \frac{1}{1024} \frac{(544 C^3 + 140 C - 986 C^2 + 1) R^{11}}{f^{10}} \right) \rho^{11} \right. \\
& + \left( -\frac{1}{128} \frac{C^2 (131 C - 32) R^9 d}{f^9} - \frac{1}{256} \frac{(106 C - 475 C^2 + 262 C^3 - 1) R^9}{f^8} \right) \rho^9 \\
& + \left( \frac{1}{32} \frac{C^2 (54 C - 23) R^7 d}{f^7} + \frac{1}{32} \frac{(-95 C^2 + 34 C + 54 C^3 - 1) R^7}{f^6} \right) \rho^7 \\
& + \left( -\frac{1}{8} \frac{C^2 (18 C - 13) R^5 d}{f^5} - \frac{1}{8} \frac{(-1 + 16 C - 35 C^2 + 18 C^3) R^5}{f^4} \right) \rho^5 \\
& + \left. \left( \frac{1}{2} \frac{C^2 (-5 + 4 C) R^3 d}{f^3} + \frac{1}{4} \frac{(4 C - 1) (2 C^2 - 4 C + 1) R^3}{f^2} \right) \rho^3 \right) \cos(\phi)^3 + \left( \left( \frac{1}{2048} \frac{C^2 (-67 + 182 C) R^{11} d}{f^{11}} + \frac{1}{12288} \frac{(-9 C + 17 + 630 C^2 - 2304 C^3 + 1092 C^4) R^{11}}{f^{10} C} \right) \right. \\
& \left. \rho^{11} \right. \\
& + \left. \left( -\frac{1}{512} \frac{C^2 (121 C - 52) R^9 d}{f^9} - \frac{1}{3072} \frac{(17 - 1527 C^3 - 12 C + 726 C^4 + 462 C^2) R^9}{f^8 C} \right) \rho^9 \right)
\end{aligned}$$

$$\begin{aligned}
& + \left( \frac{3}{128} \frac{C^2 (25C - 13) R^7 d}{f^7} + \frac{1}{384} \frac{(225 C^3 + 159 C + 1 - 468 C^2) R^7}{f^6} \right) \rho^7 \\
& + \left( -\frac{7}{16} \frac{C^2 (-2 + 3C) R^5 d}{f^5} - \frac{1}{192} \frac{(216 C + 252 C^3 - 540 C^2 - 1) R^5}{f^4} \right) \rho^5 \\
& + \left( \frac{1}{8} \frac{C^2 (20C - 19) R^3 d}{f^3} + \frac{1}{24} \frac{(-141 C^2 + 60 C^3 + 72 C + 1) R^3}{f^2} \right) \rho^3 \\
& + \left( -\frac{C^2 (-7 + 4C) R d}{f} - \frac{1}{3} (1 - 33 C^2 + 12 C + 12 C^3) R \right) \rho \Bigg) \cos(\phi) \Bigg) \psi^3 + \left( \left( \right. \right. \\
& \left. \left. \frac{31}{1024} \frac{R^{12} C^2 d}{f^{12}} + \frac{1}{2048} \frac{(62 C^2 + 4 - 91 C) R^{12}}{f^{11}} \right) \rho^{12} \right. \\
& + \left( -\frac{5}{64} \frac{R^{10} C^2 d}{f^{10}} - \frac{1}{512} \frac{(40 C^2 + 3 - 53 C) R^{10}}{f^9} \right) \rho^{10} \\
& + \left( \frac{3}{16} \frac{R^8 C^2 d}{f^8} + \frac{1}{64} \frac{(12 C - 1)(C - 1) R^8}{f^7} \right) \rho^8 \\
& + \left( -\frac{13}{32} \frac{R^6 C^2 d}{f^6} - \frac{1}{32} \frac{(1 + 13 C^2 - 12 C) R^6}{f^5} \right) \rho^6 + \left( \frac{3}{4} \frac{R^4 C^2 d}{f^4} + \frac{1}{4} \frac{C (-2 + 3C) R^4}{f^3} \right) \rho^4 \\
& + \left. \left. -\frac{R^2 C^2 d}{f^2} - \frac{1}{2} \frac{(-1 + 2 C^2) R^2}{f} \right) \rho^2 \right) \cos(\phi)^2 \\
& + \left( \frac{9}{8192} \frac{R^{12} C^2 d}{f^{12}} + \frac{1}{8192} \frac{C (9 C - 16) R^{12}}{f^{11}} \right) \rho^{12} \\
& + \left( -\frac{1}{256} \frac{R^{10} C^2 d}{f^{10}} - \frac{1}{1024} \frac{C (-7 + 4C) R^{10}}{f^9} \right) \rho^{10} \\
& + \left( \frac{7}{512} \frac{R^8 C^2 d}{f^8} + \frac{1}{512} \frac{C (-12 + 7C) R^8}{f^7} \right) \rho^8 + \left( -\frac{3}{64} \frac{R^6 C^2 d}{f^6} - \frac{1}{64} \frac{C (3 C - 5) R^6}{f^5} \right) \rho^6 \\
& + \left( \frac{5}{32} \frac{R^4 C^2 d}{f^4} + \frac{1}{32} \frac{C (-8 + 5C) R^4}{f^3} \right) \rho^4 + \left( -\frac{1}{2} \frac{R^2 C^2 d}{f^2} - \frac{1}{4} \frac{(2 C^2 - 4 C + 1) R^2}{f} \right) \rho^2 \\
& \left. + 2 f (C - 1) (C + 1) + 2 C^2 d \right) \psi^2
\end{aligned}$$

$$+ \left( -\frac{1}{1024} \frac{R^{11} \rho^{11}}{f^{10}} + \frac{1}{256} \frac{R^9 \rho^9}{f^8} - \frac{1}{64} \frac{R^7 \rho^7}{f^6} + \frac{1}{16} \frac{R^5 \rho^5}{f^4} - \frac{1}{4} \frac{R^3 \rho^3}{f^2} + 2 R \rho \right) \cos(\phi) \psi$$

> cost();

*433 additions + 3710 multiplications + 187 divisions + 13 functions*

Now calculate the Zernike series coefficients:

## **Zernike Series Calculation**

```

> verbosity := 1: time0 := time():
> ZernikeSeries( opd, 12, [Delta,psi,R,d] );
ZernikeSeries[65]:= (disk averaged OPD)
ZernikeSeries[65]:= calculating the coefficients [0, 0]...
ZernikeSeries[103]:= A[0, 0]:
```

$$\begin{aligned}
& \left( \left( \frac{17}{8192} \frac{C^2 d}{f^{14}} + \frac{1}{57344} \frac{119 C^2 - 14 - 174 C}{f^{13}} \right) R^{12} \right. \\
& + \left( -\frac{19}{3072} \frac{C^2 d}{f^{12}} - \frac{1}{3072} \frac{-3 - 25 C + 19 C^2}{f^{11}} \right) R^{10} + \left( \frac{9}{512} \frac{C^2 d}{f^{10}} + \frac{1}{2560} \frac{45 C^2 - 48 C - 10}{f^9} \right) R^8 \\
& + \left( -\frac{3}{64} \frac{C^2 d}{f^8} - \frac{1}{256} \frac{-4 - 11 C + 12 C^2}{f^7} \right) R^6 + \left( \frac{11}{96} \frac{C^2 d}{f^6} + \frac{1}{96} \frac{11 C^2 - 8 C - 6}{f^5} \right) R^4 \\
& + \left( -\frac{1}{4} \frac{C^2 d}{f^4} - \frac{1}{8} \frac{-2 + 2 C^2 - C}{f^3} \right) R^2 + \frac{1}{2} \frac{C^2 d}{f^2} + \frac{1}{2} \frac{-2 + C^2}{f} \Big) \Delta^2 + \left( \right. \\
& \left( \frac{9}{2048} \frac{C^2 d}{f^{13}} + \frac{1}{14336} \frac{63 C^2 - 2 - 93 C}{f^{12}} \right) R^{12} + \left( -\frac{41}{3072} \frac{C^2 d}{f^{11}} - \frac{1}{3072} \frac{-2 - 55 C + 41 C^2}{f^{10}} \right) R^{10} \\
& + \left( \frac{5}{128} \frac{C^2 d}{f^9} + \frac{1}{640} \frac{25 C^2 - 28 C - 2}{f^8} \right) R^8 + \left( -\frac{7}{64} \frac{C^2 d}{f^7} - \frac{1}{64} \frac{-1 - 7 C + 7 C^2}{f^6} \right) R^6 \\
& + \left( \frac{7}{24} \frac{C^2 d}{f^5} + \frac{1}{24} \frac{7 C^2 - 6 C - 2}{f^4} \right) R^4 + \left( -\frac{3}{4} \frac{C^2 d}{f^3} - \frac{1}{4} \frac{-2 + 3 C^2 - 2 C}{f^2} \right) R^2 - 4 + 2 C^2 \\
& + 2 \frac{C^2 d}{f} \Big) \Psi \Delta + \left( \left( \frac{19}{8192} \frac{C^2 d}{f^{12}} + \frac{1}{57344} \frac{133 C^2 - 198 C + 8}{f^{11}} \right) R^{12} \right. \\
& + \left( -\frac{11}{1536} \frac{C^2 d}{f^{10}} - \frac{1}{6144} \frac{44 C^2 + 3 - 60 C}{f^9} \right) R^{10} + \left( \frac{11}{512} \frac{C^2 d}{f^8} + \frac{1}{2560} \frac{-64 C + 4 + 55 C^2}{f^7} \right) R^8
\end{aligned}$$

$$\begin{aligned}
& + \left( -\frac{1}{256} \frac{(16C-1)(C-1)}{f^5} - \frac{1}{16} \frac{C^2 d}{f^6} \right) R^6 + \left( \frac{1}{96} \frac{C(-16+17C)}{f^3} + \frac{17}{96} \frac{C^2 d}{f^4} \right) R^4 \\
& + \left( -\frac{1}{2} \frac{C(C-1)}{f} - \frac{1}{2} \frac{C^2 d}{f^2} \right) R^2 + 2f(C-1)(C+1) + 2C^2 d \Bigg) \Psi^2
\end{aligned}$$

ZernikeSeries[103]: (wavefront tilt)

ZernikeSeries[103]: calculating the coefficients [1, 1]...

ZernikeSeries[310]: A[1, 1]:

$$\begin{aligned}
& \left( \left( \frac{1}{14336} \frac{C^2 (-1106 + 1259C)d}{f^{14}} + \frac{1}{14336} \frac{1259C^4 - 3212C^3 - 7C + 70 + 1611C^2}{f^{13}C} \right) R^{11} \right. \\
& + \left( -\frac{1}{1536} \frac{C^2 (-295 + 286C)d}{f^{12}} - \frac{1}{3072} \frac{770C^2 - 1483C^3 + 572C^4 - 25C + 70}{f^{11}C} \right) R^9 \\
& + \left( \frac{1}{320} \frac{C^2 (-140 + 111C)d}{f^{10}} + \frac{1}{320} \frac{146C + 15 + 111C^3 - 286C^2}{f^9} \right) R^7 \\
& + \left( -\frac{7}{64} \frac{C^2 (-8 + 5C)d}{f^8} - \frac{1}{64} \frac{35C^3 + 49C - 99C^2 + 9}{f^7} \right) R^5 \\
& + \left( \frac{1}{12} \frac{C^2 (-17 + 8C)d}{f^6} + \frac{1}{24} \frac{(4C+1)(4C^2 - 14C + 9)}{f^5} \right) R^3 \\
& + \left. \left( -\frac{1}{2} \frac{C^2 (-3 + C)d}{f^4} - \frac{1}{4} \frac{3 + 2C - 8C^2 + 2C^3}{f^3} \right) R \right) \Delta^3 + \left( \right. \\
& \left. \left( \frac{3}{7168} \frac{C^2 (743C - 475)d}{f^{13}} + \frac{1}{14336} \frac{-10342C^3 + 4458C^4 + 4290C^2 + 64C - 65}{f^{12}C} \right) R^{11} \right. \\
& + \left( -\frac{1}{1536} \frac{C^2 (1063C - 825)d}{f^{11}} - \frac{1}{3072} \frac{2244C^2 - 4992C^3 + 72C + 2126C^4 - 65}{f^{10}C} \right) R^9 \\
& + \left( \frac{9}{320} \frac{C^2 (49C - 48)d}{f^9} + \frac{3}{320} \frac{2 - 342C^2 + 147C^3 + 164C}{f^8} \right) R^7 \\
& + \left( -\frac{3}{64} \frac{C^2 (51C - 65)d}{f^7} - \frac{1}{64} \frac{7 + 191C + 153C^3 - 388C^2}{f^6} \right) R^5 \\
& + \left. \left( \frac{1}{12} \frac{C^2 (40C - 69)d}{f^5} + \frac{1}{24} \frac{80C^3 - 230C^2 + 104C + 13}{f^4} \right) R^3 \right)
\end{aligned}$$

$$\begin{aligned}
& + \left( -\frac{3}{2} \frac{C^2 (2C-5)d}{f^3} - \frac{3}{4} \frac{4C^3 + 4C + 3 - 14C^2}{f^2} \right) R \right) \Psi \Delta^2 + \left( \right. \\
& \left. \left( \frac{5}{14336} \frac{C^2 (-420 + 1039C)d}{f^{12}} + \frac{1}{14336} \frac{-12C - 65 - 10868C^3 + 5195C^4 + 3329C^2}{f^{11}C} \right) R^{11} \right. \\
& + \left( -\frac{1}{1536} \frac{C^2 (1291C - 683)d}{f^{10}} - \frac{1}{3072} \frac{-5453C^3 + 11C - 65 + 1962C^2 + 2582C^4}{f^9C} \right) R^9 \\
& + \left( \frac{1}{320} \frac{C^2 (567C - 404)d}{f^8} + \frac{1}{320} \frac{567C^3 - 17 + 507C - 1185C^2}{f^7} \right) R^7 \\
& + \left( -\frac{1}{64} \frac{C^2 (-209 + 214C)d}{f^6} - \frac{1}{128} \frac{460C + 428C^3 - 17 - 974C^2}{f^5} \right) R^5 \\
& + \left( \frac{2}{3} \frac{C^2 (8C - 11)d}{f^4} + \frac{1}{24} \frac{-3 + 128C^3 - 328C^2 + 156C}{f^3} \right) R^3 \\
& + \left. \left( -\frac{1}{2} \frac{C^2 (12C - 25)d}{f^2} - \frac{1}{2} \frac{12C^3 + 4 + 12C - 37C^2}{f} \right) R \right) \Psi^2 - \frac{1}{3584} \frac{R^{11}}{f^{11}} + \frac{1}{768} \frac{R^9}{f^9} \\
& - \frac{1}{160} \frac{R^7}{f^7} + \frac{1}{32} \frac{R^5}{f^5} - \frac{1}{6} \frac{R^3}{f^3} + \frac{R}{f} \right) \Delta + \left( \right. \\
& \left. \left( \frac{1}{3584} \frac{C^2 (-86 + 499C)d}{f^{11}} + \frac{1}{43008} \frac{-11178C^3 + 17 + 1890C^2 + 5988C^4}{f^{10}C} \right) R^{11} \right. \\
& + \left( -\frac{1}{768} \frac{C^2 (-74 + 257C)d}{f^9} - \frac{1}{9216} \frac{3084C^4 + 1416C^2 - 5802C^3 + 17 - 21C}{f^8C} \right) R^9 \\
& + \left( \frac{3}{320} \frac{C^2 (-36 + 79C)d}{f^7} + \frac{1}{960} \frac{711C^3 - 8 - 1323C^2 + 465C}{f^6} \right) R^7 \\
& + \left( -\frac{1}{64} \frac{C^2 (96C - 67)d}{f^5} - \frac{1}{384} \frac{-19 + 504C - 1170C^2 + 576C^3}{f^4} \right) R^5 \\
& + \left( \frac{1}{6} \frac{C^2 (-17 + 16C)d}{f^3} + \frac{1}{72} \frac{-444C^2 + 216C + 192C^3 - 7}{f^2} \right) R^3 \\
& + \left. \left( -\frac{C^2 (-7 + 4C)d}{f} + 11C^2 - 4C - 4C^3 - \frac{1}{3} \right) R \right) \Psi^3
\end{aligned}$$

$$+ \left( -\frac{1}{3584} \frac{R^{11}}{f^{10}} + \frac{1}{768} \frac{R^9}{f^8} - \frac{1}{160} \frac{R^7}{f^6} + \frac{1}{32} \frac{R^5}{f^4} - \frac{1}{6} \frac{R^3}{f^2} + 2R \right) \Psi$$

ZernikeSeries[310]: (defocus)

ZernikeSeries[311]: calculating the coefficients [2, 0]...

ZernikeSeries[334]: A[2, 0]:

$$\begin{aligned} & \left( \left( \frac{153}{32768} \frac{C^2 d}{f^{14}} + \frac{9}{229376} \frac{119 C^2 - 14 - 174 C}{f^{13}} \right) R^{12} \right. \\ & + \left( -\frac{95}{7168} \frac{C^2 d}{f^{12}} - \frac{5}{7168} \frac{-3 - 25 C + 19 C^2}{f^{11}} \right) R^{10} + \left( \frac{9}{256} \frac{C^2 d}{f^{10}} + \frac{1}{1280} \frac{45 C^2 - 48 C - 10}{f^9} \right) R^8 \\ & + \left( -\frac{27}{320} \frac{C^2 d}{f^8} - \frac{9}{1280} \frac{-4 - 11 C + 12 C^2}{f^7} \right) R^6 + \left( \frac{11}{64} \frac{C^2 d}{f^6} + \frac{1}{64} \frac{11 C^2 - 8 C - 6}{f^5} \right) R^4 \\ & + \left. \left( -\frac{1}{4} \frac{C^2 d}{f^4} - \frac{1}{8} \frac{-2 + 2 C^2 - C}{f^3} \right) R^2 \right) \Delta^2 + \left( \left( \frac{81}{8192} \frac{C^2 d}{f^{13}} + \frac{9}{57344} \frac{63 C^2 - 2 - 93 C}{f^{12}} \right) R^{12} \right. \\ & + \left( -\frac{205}{7168} \frac{C^2 d}{f^{11}} - \frac{5}{7168} \frac{-2 - 55 C + 41 C^2}{f^{10}} \right) R^{10} + \left( \frac{5}{64} \frac{C^2 d}{f^9} + \frac{1}{320} \frac{25 C^2 - 28 C - 2}{f^8} \right) R^8 \\ & + \left( -\frac{63}{320} \frac{C^2 d}{f^7} - \frac{9}{320} \frac{-1 - 7 C + 7 C^2}{f^6} \right) R^6 + \left( \frac{7}{16} \frac{C^2 d}{f^5} + \frac{1}{16} \frac{7 C^2 - 6 C - 2}{f^4} \right) R^4 \\ & + \left. \left( -\frac{3}{4} \frac{C^2 d}{f^3} - \frac{1}{4} \frac{-2 + 3 C^2 - 2 C}{f^2} \right) R^2 \right) \Psi \Delta + \left( \right. \\ & \left( \frac{171}{32768} \frac{C^2 d}{f^{12}} + \frac{9}{229376} \frac{133 C^2 - 198 C + 8}{f^{11}} \right) R^{12} \\ & + \left( -\frac{55}{3584} \frac{C^2 d}{f^{10}} - \frac{5}{14336} \frac{44 C^2 + 3 - 60 C}{f^9} \right) R^{10} + \left( \frac{11}{256} \frac{C^2 d}{f^8} + \frac{1}{1280} \frac{-64 C + 4 + 55 C^2}{f^7} \right) R^8 \\ & + \left( -\frac{9}{80} \frac{C^2 d}{f^6} - \frac{9}{1280} \frac{(16 C - 1)(C - 1)}{f^5} \right) R^6 + \left( \frac{17}{64} \frac{C^2 d}{f^4} + \frac{1}{64} \frac{C(-16 + 17 C)}{f^3} \right) R^4 \\ & + \left. \left( -\frac{1}{2} \frac{C(C - 1)}{f} - \frac{1}{2} \frac{C^2 d}{f^2} \right) R^2 \right) \Psi^2 \end{aligned}$$

ZernikeSeries[334]: (astigmatism)

ZernikeSeries[334]: calculating the coefficients [2, 2]...

ZernikeSeries[563]: A[2, 2]:

$$\begin{aligned}
& \left( \left( \frac{21}{4096} \frac{C^2 d}{f^{14}} + \frac{3}{32768} \frac{56 C^2 - 6 - 81 C}{f^{13}} \right) R^{12} + \left( -\frac{15}{1024} \frac{C^2 d}{f^{12}} - \frac{15}{7168} \frac{7 C^2 - 1 - 9 C}{f^{11}} \right) R^{10} \right. \\
& + \left( \frac{5}{128} \frac{C^2 d}{f^{10}} + \frac{1}{128} \frac{5 C^2 - 5 C - 1}{f^9} \right) R^8 + \left( -\frac{3}{32} \frac{C^2 d}{f^8} - \frac{3}{320} \frac{-3 + 10 C^2 - 8 C}{f^7} \right) R^6 \\
& + \left. \left( \frac{3}{16} \frac{C^2 d}{f^6} + \frac{3}{32} \frac{(2 C + 1)(C - 1)}{f^5} \right) R^4 + \left( -\frac{1}{4} \frac{C^2 d}{f^4} - \frac{1}{4} \frac{(C - 1)(C + 1)}{f^3} \right) R^2 \right) \Delta^2 + \left( \right. \\
& \left( \frac{177}{16384} \frac{C^2 d}{f^{13}} + \frac{3}{16384} \frac{-86 C - 1 + 59 C^2}{f^{12}} \right) R^{12} \\
& + \left. \left( -\frac{225}{7168} \frac{C^2 d}{f^{11}} - \frac{3}{7168} \frac{75 C^2 - 98 C - 2}{f^{10}} \right) R^{10} + \left( \frac{11}{128} \frac{C^2 d}{f^9} + \frac{1}{256} \frac{-1 - 23 C + 22 C^2}{f^8} \right) R^8 \right. \\
& + \left. \left( -\frac{69}{320} \frac{C^2 d}{f^7} - \frac{3}{320} \frac{23 C^2 - 20 C - 2}{f^6} \right) R^6 + \left( \frac{15}{32} \frac{C^2 d}{f^5} + \frac{3}{32} \frac{5 C^2 - 1 - 3 C}{f^4} \right) R^4 \right. \\
& + \left. \left( -\frac{3}{4} \frac{C^2 d}{f^3} - \frac{1}{4} \frac{3 C^2 - 2}{f^2} \right) R^2 \right) \Psi \Delta + \left( \left( \frac{93}{16384} \frac{C^2 d}{f^{12}} + \frac{3}{32768} \frac{62 C^2 + 4 - 91 C}{f^{11}} \right) R^{12} \right. \\
& + \left. \left( -\frac{15}{896} \frac{C^2 d}{f^{10}} - \frac{3}{7168} \frac{40 C^2 + 3 - 53 C}{f^9} \right) R^{10} + \left( \frac{3}{64} \frac{C^2 d}{f^8} + \frac{1}{256} \frac{(12 C - 1)(C - 1)}{f^7} \right) R^8 \right. \\
& + \left. \left( -\frac{39}{320} \frac{C^2 d}{f^6} - \frac{3}{320} \frac{1 + 13 C^2 - 12 C}{f^5} \right) R^6 + \left( \frac{9}{32} \frac{C^2 d}{f^4} + \frac{3}{32} \frac{C(-2 + 3 C)}{f^3} \right) R^4 \right. \\
& + \left. \left( -\frac{1}{2} \frac{C^2 d}{f^2} - \frac{1}{4} \frac{-1 + 2 C^2}{f} \right) R^2 \right) \Psi^2
\end{aligned}$$

ZernikeSeries[563]: (coma)

ZernikeSeries[563]: calculating the coefficients [3, 1]...

ZernikeSeries[651]: A[3, 1]:

$$\begin{aligned}
& \left( \left( \frac{5}{57344} \frac{C^2 (-1106 + 1259 C) d}{f^{14}} + \frac{5}{57344} \frac{1259 C^4 - 3212 C^3 - 7 C + 70 + 1611 C^2}{f^{13} C} \right) R^{11} \right. \\
& + \left. \left( -\frac{1}{1344} \frac{C^2 (-295 + 286 C) d}{f^{12}} - \frac{1}{2688} \frac{770 C^2 - 1483 C^3 + 572 C^4 - 25 C + 70}{f^{11} C} \right) R^9 \right. \\
& + \left. \left( \frac{1}{320} \frac{C^2 (-140 + 111 C) d}{f^{10}} + \frac{1}{320} \frac{146 C + 15 + 111 C^3 - 286 C^2}{f^9} \right) R^7 \right)
\end{aligned}$$

$$\begin{aligned}
& + \left( -\frac{7}{80} \frac{C^2 (-8+5C)d}{f^8} - \frac{1}{80} \frac{35C^3 + 49C - 99C^2 + 9}{f^7} \right) R^5 \\
& + \left( \frac{1}{24} \frac{C^2 (-17+8C)d}{f^6} + \frac{1}{48} \frac{(4C+1)(4C^2 - 14C + 9)}{f^5} \right) R^3 \Delta^3 + \left( \right. \\
& \left. \left( \frac{15}{28672} \frac{C^2 (743C - 475)d}{f^{13}} + \frac{5}{57344} \frac{-10342C^3 + 4458C^4 + 4290C^2 + 64C - 65}{f^{12}C} \right) R^{11} \right. \\
& + \left( -\frac{1}{1344} \frac{C^2 (1063C - 825)d}{f^{11}} - \frac{1}{2688} \frac{2244C^2 - 4992C^3 + 72C + 2126C^4 - 65}{f^{10}C} \right) R^9 \\
& + \left( \frac{9}{320} \frac{C^2 (49C - 48)d}{f^9} + \frac{3}{320} \frac{2 - 342C^2 + 147C^3 + 164C}{f^8} \right) R^7 \\
& + \left( -\frac{3}{80} \frac{C^2 (51C - 65)d}{f^7} - \frac{1}{80} \frac{7 + 191C + 153C^3 - 388C^2}{f^6} \right) R^5 \\
& + \left( \frac{1}{24} \frac{C^2 (40C - 69)d}{f^5} + \frac{1}{48} \frac{80C^3 - 230C^2 + 104C + 13}{f^4} \right) R^3 \Psi \Delta^2 + \left( \right. \\
& \left. \left( \frac{25}{57344} \frac{C^2 (-420 + 1039C)d}{f^{12}} + \frac{5}{57344} \frac{-12C - 65 - 10868C^3 + 5195C^4 + 3329C^2}{f^{11}C} \right) R^{11} \right. \\
& + \left( -\frac{1}{1344} \frac{C^2 (1291C - 683)d}{f^{10}} - \frac{1}{2688} \frac{-5453C^3 + 11C - 65 + 1962C^2 + 2582C^4}{f^9C} \right) R^9 \\
& + \left( \frac{1}{320} \frac{C^2 (567C - 404)d}{f^8} + \frac{1}{320} \frac{567C^3 - 17 + 507C - 1185C^2}{f^7} \right) R^7 \\
& + \left( -\frac{1}{80} \frac{C^2 (-209 + 214C)d}{f^6} - \frac{1}{160} \frac{460C + 428C^3 - 17 - 974C^2}{f^5} \right) R^5 \\
& + \left( \frac{1}{3} \frac{C^2 (8C - 11)d}{f^4} + \frac{1}{48} \frac{-3 + 128C^3 - 328C^2 + 156C}{f^3} \right) R^3 \Psi^2 - \frac{5}{14336} \frac{R^{11}}{f^{11}} + \frac{1}{672} \frac{R^9}{f^9} \\
& - \frac{1}{160} \frac{R^7}{f^7} + \frac{1}{40} \frac{R^5}{f^5} - \frac{1}{12} \frac{R^3}{f^3} \right) \Delta + \left( \right. \\
& \left. \left( \frac{5}{14336} \frac{C^2 (-86 + 499C)d}{f^{11}} + \frac{5}{172032} \frac{-11178C^3 + 17 + 1890C^2 + 5988C^4}{f^{10}C} \right) R^{11} \right)
\end{aligned}$$

$$\begin{aligned}
& + \left( -\frac{1}{672} \frac{C^2 (-74 + 257 C) d}{f^9} - \frac{1}{8064} \frac{3084 C^4 + 1416 C^2 - 5802 C^3 + 17 - 21 C}{f^8 C} \right) R^9 \\
& + \left( \frac{3}{320} \frac{C^2 (-36 + 79 C) d}{f^7} + \frac{1}{960} \frac{711 C^3 - 8 - 1323 C^2 + 465 C}{f^6} \right) R^7 \\
& + \left( -\frac{1}{80} \frac{C^2 (96 C - 67) d}{f^5} - \frac{1}{480} \frac{-19 + 504 C - 1170 C^2 + 576 C^3}{f^4} \right) R^5 \\
& + \left( \frac{1}{12} \frac{C^2 (-17 + 16 C) d}{f^3} + \frac{1}{144} \frac{-444 C^2 + 216 C + 192 C^3 - 7}{f^2} \right) R^3 \Bigg) \Psi^3 \\
& + \left( -\frac{5}{14336} \frac{R^{11}}{f^{10}} + \frac{1}{672} \frac{R^9}{f^8} - \frac{1}{160} \frac{R^7}{f^6} + \frac{1}{40} \frac{R^5}{f^4} - \frac{1}{12} \frac{R^3}{f^2} \right) \Psi
\end{aligned}$$

ZernikeSeries[651]: (trefoil)

ZernikeSeries[651]: calculating the coefficients [3, 3]...

ZernikeSeries[890]: A[3, 3]:

$$\begin{aligned}
& \left( \frac{1}{8192} \frac{C^2 (-280 + 359 C) d}{f^{14}} + \frac{1}{8192} \frac{-878 C^3 + C + 359 C^4 + 14 + 405 C^2}{f^{13} C} \right) R^{11} \\
& + \left( -\frac{1}{128} \frac{C^2 (-10 + 11 C) d}{f^{12}} - \frac{1}{896} \frac{77 C^4 + 7 - 2 C - 189 C^3 + 90 C^2}{f^{11} C} \right) R^9 \\
& + \left( \frac{1}{64} \frac{C^2 (9 C - 10) d}{f^{10}} + \frac{1}{192} \frac{30 C + 3 - 64 C^2 + 27 C^3}{f^9} \right) R^7 \\
& + \left( -\frac{1}{40} \frac{C^2 (-10 + 7 C) d}{f^8} - \frac{1}{80} \frac{14 C^3 + 16 C - 36 C^2 + 3}{f^7} \right) R^5 \\
& + \left( \frac{1}{8} \frac{C^2 (C - 2) d}{f^6} + \frac{1}{16} \frac{1 + 2 C^3 - 6 C^2 + 2 C}{f^5} \right) R^3 \Bigg) \Delta^3 + \Bigg( \\
& \left( \frac{1}{2048} \frac{C^2 (313 C - 165) d}{f^{13}} + \frac{1}{8192} \frac{1252 C^4 + 996 C^2 - 2754 C^3 - 12 + 7 C}{f^{12} C} \right) R^{11} \\
& + \left( -\frac{1}{896} \frac{C^2 (-180 + 281 C) d}{f^{11}} - \frac{1}{1792} \frac{-12 + 488 C^2 + 10 C - 1233 C^3 + 562 C^4}{f^{10} C} \right) R^9 \\
& + \left( \frac{1}{192} \frac{C^2 (-86 + 105 C) d}{f^9} + \frac{1}{192} \frac{C (-221 C + 105 C^2 + 97)}{f^8} \right) R^7
\end{aligned}$$

$$\begin{aligned}
& + \left( -\frac{3}{40} \frac{C^2 (-11 + 10 C) d}{f^7} - \frac{1}{80} \frac{1 + 60 C^3 + 62 C - 136 C^2}{f^6} \right) R^5 \\
& + \left( \frac{1}{8} \frac{C^2 (-8 + 5 C) d}{f^5} + \frac{1}{16} \frac{-26 C^2 + 10 C + 1 + 10 C^3}{f^4} \right) R^3 \Bigg) \Psi \Delta^2 + \Bigg( \\
& \left( \frac{3}{8192} \frac{C^2 (-150 + 479 C) d}{f^{12}} + \frac{1}{8192} \frac{-2862 C^3 + 2 C - 21 + 731 C^2 + 1437 C^4}{f^{11} C} \right) R^{11} \\
& + \left( -\frac{1}{896} \frac{C^2 (-142 + 335 C) d}{f^{10}} - \frac{1}{1792} \frac{414 C^2 + 670 C^4 + 8 C - 21 - 1330 C^3}{f^9 C} \right) R^9 \\
& + \left( \frac{1}{192} \frac{C^2 (-79 + 132 C) d}{f^8} + \frac{1}{192} \frac{(2 C - 1) (66 C^2 - 93 C + 4)}{f^7} \right) R^7 \\
& + \left( -\frac{1}{40} \frac{C^2 (-36 + 41 C) d}{f^6} - \frac{1}{40} \frac{-85 C^2 + 41 C^3 + 39 C - 2}{f^5} \right) R^5 \\
& + \left( \frac{1}{8} \frac{C^2 (8 C - 11) d}{f^4} + \frac{1}{16} \frac{-38 C^2 - 1 + 16 C + 16 C^3}{f^3} \right) R^3 \Bigg) \Psi^2 \Delta + \Bigg( \\
& \left( \frac{1}{4096} \frac{C^2 (272 C - 35) d}{f^{11}} + \frac{1}{8192} \frac{544 C^3 + 140 C - 986 C^2 + 1}{f^{10}} \right) R^{11} \\
& + \left( -\frac{1}{896} \frac{C^2 (131 C - 32) d}{f^9} - \frac{1}{1792} \frac{106 C - 475 C^2 + 262 C^3 - 1}{f^8} \right) R^9 \\
& + \left( \frac{1}{192} \frac{C^2 (54 C - 23) d}{f^7} + \frac{1}{192} \frac{-95 C^2 + 34 C + 54 C^3 - 1}{f^6} \right) R^7 \\
& + \left( -\frac{1}{40} \frac{C^2 (18 C - 13) d}{f^5} - \frac{1}{40} \frac{-1 + 16 C - 35 C^2 + 18 C^3}{f^4} \right) R^5 \\
& + \left( \frac{1}{8} \frac{C^2 (-5 + 4 C) d}{f^3} + \frac{1}{16} \frac{(4 C - 1) (2 C^2 - 4 C + 1)}{f^2} \right) R^3 \Bigg) \Psi^3
\end{aligned}$$

ZernikeSeries[890]: (spherical)

ZernikeSeries[890]: calculating the coefficients [4, 0]...

ZernikeSeries[949]: A[4, 0]:

$$\left( \left( \frac{425}{98304} \frac{C^2 d}{f^{14}} + \frac{25}{688128} \frac{119 C^2 - 14 - 174 C}{f^{13}} \right) R^{12} \right)$$

$$\begin{aligned}
& + \left( -\frac{475}{43008} \frac{C^2 d}{f^{12}} - \frac{25}{43008} \frac{-3 - 25 C + 19 C^2}{f^{11}} \right) R^{10} \\
& + \left( \frac{45}{1792} \frac{C^2 d}{f^{10}} + \frac{1}{1792} \frac{45 C^2 - 48 C - 10}{f^9} \right) R^8 + \left( -\frac{3}{64} \frac{C^2 d}{f^8} - \frac{1}{256} \frac{-4 - 11 C + 12 C^2}{f^7} \right) R^6 \\
& + \left( \frac{11}{192} \frac{C^2 d}{f^6} + \frac{1}{192} \frac{11 C^2 - 8 C - 6}{f^5} \right) R^4 \right) \Delta^2 + \left( \right. \\
& \left. \left( \frac{75}{8192} \frac{C^2 d}{f^{13}} + \frac{25}{172032} \frac{63 C^2 - 2 - 93 C}{f^{12}} \right) R^{12} \right. \\
& + \left( -\frac{1025}{43008} \frac{C^2 d}{f^{11}} - \frac{25}{43008} \frac{-2 - 55 C + 41 C^2}{f^{10}} \right) R^{10} + \left( \frac{25}{448} \frac{C^2 d}{f^9} + \frac{1}{448} \frac{25 C^2 - 28 C - 2}{f^8} \right) R^8 \\
& + \left( -\frac{7}{64} \frac{C^2 d}{f^7} - \frac{1}{64} \frac{-1 - 7 C + 7 C^2}{f^6} \right) R^6 + \left( \frac{7}{48} \frac{C^2 d}{f^5} + \frac{1}{48} \frac{7 C^2 - 6 C - 2}{f^4} \right) R^4 \right) \Psi \Delta + \left( \right. \\
& \left. \left( \frac{475}{98304} \frac{C^2 d}{f^{12}} + \frac{25}{688128} \frac{133 C^2 - 198 C + 8}{f^{11}} \right) R^{12} \right. \\
& + \left( -\frac{275}{21504} \frac{C^2 d}{f^{10}} - \frac{25}{86016} \frac{44 C^2 + 3 - 60 C}{f^9} \right) R^{10} \\
& + \left( \frac{55}{1792} \frac{C^2 d}{f^8} + \frac{1}{1792} \frac{-64 C + 4 + 55 C^2}{f^7} \right) R^8 + \left( -\frac{1}{256} \frac{(16 C - 1)(C - 1)}{f^5} - \frac{1}{16} \frac{C^2 d}{f^6} \right) R^6 \\
& + \left. \left( \frac{17}{192} \frac{C^2 d}{f^4} + \frac{1}{192} \frac{C(-16 + 17 C)}{f^3} \right) R^4 \right) \Psi^2
\end{aligned}$$

ZernikeSeries[949]: calculating the coefficients [4, 2]...

ZernikeSeries[1064]: A[4, 2]:

$$\begin{aligned}
& \left( \left( \frac{175}{36864} \frac{C^2 d}{f^{14}} + \frac{25}{294912} \frac{56 C^2 - 6 - 81 C}{f^{13}} \right) R^{12} \right. \\
& + \left( -\frac{25}{2048} \frac{C^2 d}{f^{12}} - \frac{25}{14336} \frac{7 C^2 - 1 - 9 C}{f^{11}} \right) R^{10} + \left( \frac{25}{896} \frac{C^2 d}{f^{10}} + \frac{5}{896} \frac{5 C^2 - 5 C - 1}{f^9} \right) R^8 \\
& + \left. \left( -\frac{5}{96} \frac{C^2 d}{f^8} - \frac{1}{192} \frac{-3 + 10 C^2 - 8 C}{f^7} \right) R^6 + \left( \frac{1}{16} \frac{C^2 d}{f^6} + \frac{1}{32} \frac{(2 C + 1)(C - 1)}{f^5} \right) R^4 \right) \Delta^2 + \left( \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{1475}{147456} \frac{C^2 d}{f^{13}} + \frac{25}{147456} \frac{-86 C - 1 + 59 C^2}{f^{12}} \right) R^{12} \\
& + \left( -\frac{375}{14336} \frac{C^2 d}{f^{11}} - \frac{5}{14336} \frac{75 C^2 - 98 C - 2}{f^{10}} \right) R^{10} \\
& + \left( \frac{55}{896} \frac{C^2 d}{f^9} + \frac{5}{1792} \frac{-1 - 23 C + 22 C^2}{f^8} \right) R^8 + \left( -\frac{23}{192} \frac{C^2 d}{f^7} - \frac{1}{192} \frac{23 C^2 - 20 C - 2}{f^6} \right) R^6 \\
& + \left( \frac{5}{32} \frac{C^2 d}{f^5} + \frac{1}{32} \frac{5 C^2 - 1 - 3 C}{f^4} \right) R^4 \right) \Psi \Delta + \left( \right. \\
& \left. \frac{775}{147456} \frac{C^2 d}{f^{12}} + \frac{25}{294912} \frac{62 C^2 + 4 - 91 C}{f^{11}} \right) R^{12} \\
& + \left( -\frac{25}{1792} \frac{C^2 d}{f^{10}} - \frac{5}{14336} \frac{40 C^2 + 3 - 53 C}{f^9} \right) R^{10} \\
& + \left( \frac{15}{448} \frac{C^2 d}{f^8} + \frac{5}{1792} \frac{(12 C - 1)(C - 1)}{f^7} \right) R^8 + \left( -\frac{13}{192} \frac{C^2 d}{f^6} - \frac{1}{192} \frac{1 + 13 C^2 - 12 C}{f^5} \right) R^6 \\
& \left. + \left( \frac{3}{32} \frac{C^2 d}{f^4} + \frac{1}{32} \frac{C(-2 + 3 C)}{f^3} \right) R^4 \right) \Psi^2
\end{aligned}$$

ZernikeSeries[1064]: calculating the coefficients [4, 4]...

ZernikeSeries[1418]: calculating the coefficients [5, 1]...

ZernikeSeries[1633]: A[5, 1]:

$$\begin{aligned}
& \left( \left( \frac{5}{86016} \frac{C^2 (-1106 + 1259 C) d}{f^{14}} + \frac{5}{86016} \frac{1259 C^4 - 3212 C^3 - 7 C + 70 + 1611 C^2}{f^{13} C} \right) R^{11} \right. \\
& + \left( -\frac{3}{7168} \frac{C^2 (-295 + 286 C) d}{f^{12}} - \frac{3}{14336} \frac{770 C^2 - 1483 C^3 + 572 C^4 - 25 C + 70}{f^{11} C} \right) R^9 \\
& + \left( \frac{3}{2240} \frac{C^2 (-140 + 111 C) d}{f^{10}} + \frac{3}{2240} \frac{146 C + 15 + 111 C^3 - 286 C^2}{f^9} \right) R^7 \\
& + \left. \left( -\frac{7}{320} \frac{C^2 (-8 + 5 C) d}{f^8} - \frac{1}{320} \frac{35 C^3 + 49 C - 99 C^2 + 9}{f^7} \right) R^5 \right) \Delta^3 + \left( \right. \\
& \left. \left( \frac{5}{14336} \frac{C^2 (743 C - 475) d}{f^{13}} + \frac{5}{86016} \frac{-10342 C^3 + 4458 C^4 + 4290 C^2 + 64 C - 65}{f^{12} C} \right) R^{11} \right)
\end{aligned}$$

$$\begin{aligned}
& + \left( -\frac{3}{7168} \frac{C^2 (1063 C - 825) d}{f^{11}} - \frac{3}{14336} \frac{2244 C^2 - 4992 C^3 + 72 C + 2126 C^4 - 65}{f^{10} C} \right) R^9 \\
& + \left( \frac{27}{2240} \frac{C^2 (49 C - 48) d}{f^9} + \frac{9}{2240} \frac{2 - 342 C^2 + 147 C^3 + 164 C}{f^8} \right) R^7 \\
& + \left( -\frac{3}{320} \frac{C^2 (51 C - 65) d}{f^7} - \frac{1}{320} \frac{7 + 191 C + 153 C^3 - 388 C^2}{f^6} \right) R^5 \Big) \Psi \Delta^2 + \left( \left( \right. \right. \\
& \left. \left. \frac{25}{86016} \frac{C^2 (-420 + 1039 C) d}{f^{12}} + \frac{5}{86016} \frac{-12 C - 65 - 10868 C^3 + 5195 C^4 + 3329 C^2}{f^{11} C} \right) R^{11} \right. \\
& + \left( -\frac{3}{7168} \frac{C^2 (1291 C - 683) d}{f^{10}} - \frac{3}{14336} \frac{-5453 C^3 + 11 C - 65 + 1962 C^2 + 2582 C^4}{f^9 C} \right) R^9 \\
& + \left( \frac{3}{2240} \frac{C^2 (567 C - 404) d}{f^8} + \frac{3}{2240} \frac{567 C^3 - 17 + 507 C - 1185 C^2}{f^7} \right) R^7 \\
& + \left( -\frac{1}{320} \frac{C^2 (-209 + 214 C) d}{f^6} - \frac{1}{640} \frac{460 C + 428 C^3 - 17 - 974 C^2}{f^5} \right) R^5 \Big) \Psi^2 - \frac{5}{21504} \frac{R^{11}}{f^{11}} \\
& + \frac{3}{3584} \frac{R^9}{f^9} - \frac{3}{1120} \frac{R^7}{f^7} + \frac{1}{160} \frac{R^5}{f^5} \Big) \Delta + \left( \right. \\
& \left. \left( \frac{5}{21504} \frac{C^2 (-86 + 499 C) d}{f^{11}} + \frac{5}{258048} \frac{-11178 C^3 + 17 + 1890 C^2 + 5988 C^4}{f^{10} C} \right) R^{11} \right. \\
& + \left( -\frac{3}{3584} \frac{C^2 (-74 + 257 C) d}{f^9} - \frac{1}{14336} \frac{3084 C^4 + 1416 C^2 - 5802 C^3 + 17 - 21 C}{f^8 C} \right) R^9 \\
& + \left( \frac{9}{2240} \frac{C^2 (-36 + 79 C) d}{f^7} + \frac{1}{2240} \frac{711 C^3 - 8 - 1323 C^2 + 465 C}{f^6} \right) R^7 \\
& + \left( -\frac{1}{320} \frac{C^2 (96 C - 67) d}{f^5} - \frac{1}{1920} \frac{-19 + 504 C - 1170 C^2 + 576 C^3}{f^4} \right) R^5 \Big) \Psi^3 \\
& + \left( -\frac{5}{21504} \frac{R^{11}}{f^{10}} + \frac{3}{3584} \frac{R^9}{f^8} - \frac{3}{1120} \frac{R^7}{f^6} + \frac{1}{160} \frac{R^5}{f^4} \right) \Psi
\end{aligned}$$

ZernikeSeries[1634]: calculating the coefficients [5, 3]...  
ZernikeSeries[1991]: A[5, 3]:

$$\begin{aligned}
& \left( \left( \frac{1}{12288} \frac{C^2 (-280 + 359 C) d}{f^{14}} + \frac{1}{12288} \frac{-878 C^3 + C + 359 C^4 + 14 + 405 C^2}{f^{13} C} \right) R^{11} \right. \\
& + \left. \left( -\frac{9}{2048} \frac{C^2 (-10 + 11 C) d}{f^{12}} - \frac{9}{14336} \frac{77 C^4 + 7 - 2 C - 189 C^3 + 90 C^2}{f^{11} C} \right) R^9 \right. \\
& + \left. \left( \frac{3}{448} \frac{C^2 (9 C - 10) d}{f^{10}} + \frac{1}{448} \frac{30 C + 3 - 64 C^2 + 27 C^3}{f^9} \right) R^7 \right. \\
& + \left. \left( -\frac{1}{160} \frac{C^2 (-10 + 7 C) d}{f^8} - \frac{1}{320} \frac{14 C^3 + 16 C - 36 C^2 + 3}{f^7} \right) R^5 \right) \Delta^3 + \left( \right. \\
& \left. \left( \frac{1}{3072} \frac{C^2 (313 C - 165) d}{f^{13}} + \frac{1}{12288} \frac{1252 C^4 + 996 C^2 - 2754 C^3 - 12 + 7 C}{f^{12} C} \right) R^{11} \right. \\
& + \left. \left( -\frac{9}{14336} \frac{C^2 (-180 + 281 C) d}{f^{11}} - \frac{9}{28672} \frac{-12 + 488 C^2 + 10 C - 1233 C^3 + 562 C^4}{f^{10} C} \right) R^9 \right. \\
& + \left. \left( \frac{1}{448} \frac{C^2 (-86 + 105 C) d}{f^9} + \frac{1}{448} \frac{C (-221 C + 105 C^2 + 97)}{f^8} \right) R^7 \right. \\
& + \left. \left( -\frac{3}{160} \frac{C^2 (-11 + 10 C) d}{f^7} - \frac{1}{320} \frac{1 + 60 C^3 + 62 C - 136 C^2}{f^6} \right) R^5 \right) \Psi \Delta^2 + \left( \right. \\
& \left. \left( \frac{1}{4096} \frac{C^2 (-150 + 479 C) d}{f^{12}} + \frac{1}{12288} \frac{-2862 C^3 + 2 C - 21 + 731 C^2 + 1437 C^4}{f^{11} C} \right) R^{11} \right. \\
& + \left. \left( -\frac{9}{14336} \frac{C^2 (-142 + 335 C) d}{f^{10}} - \frac{9}{28672} \frac{414 C^2 + 670 C^4 + 8 C - 21 - 1330 C^3}{f^9 C} \right) R^9 \right. \\
& + \left. \left( \frac{1}{448} \frac{C^2 (-79 + 132 C) d}{f^8} + \frac{1}{448} \frac{(2 C - 1)(66 C^2 - 93 C + 4)}{f^7} \right) R^7 \right. \\
& + \left. \left( -\frac{1}{160} \frac{C^2 (-36 + 41 C) d}{f^6} - \frac{1}{160} \frac{-85 C^2 + 41 C^3 + 39 C - 2}{f^5} \right) R^5 \right) \Psi^2 \Delta + \left( \right. \\
& \left. \left( \frac{1}{6144} \frac{C^2 (272 C - 35) d}{f^{11}} + \frac{1}{12288} \frac{544 C^3 + 140 C - 986 C^2 + 1}{f^{10}} \right) R^{11} \right. \\
& + \left. \left( -\frac{9}{14336} \frac{C^2 (131 C - 32) d}{f^9} - \frac{9}{28672} \frac{106 C - 475 C^2 + 262 C^3 - 1}{f^8} \right) R^9 \right)
\end{aligned}$$

$$+ \left( \frac{1}{448} \frac{C^2 (54 C - 23) d}{f^7} + \frac{1}{448} \frac{-95 C^2 + 34 C + 54 C^3 - 1}{f^6} \right) R^7 \\ + \left( -\frac{1}{160} \frac{C^2 (18 C - 13) d}{f^5} - \frac{1}{160} \frac{-1 + 16 C - 35 C^2 + 18 C^3}{f^4} \right) R^5 \Bigg) \Psi^3$$

ZernikeSeries[1992]: calculating the coefficients [5, 5]...

ZernikeSeries[2658]: calculating the coefficients [6, 0]...

ZernikeSeries[2811]: A[6, 0]:

$$\left( \left( \frac{119}{49152} \frac{C^2 d}{f^{14}} + \frac{1}{49152} \frac{119 C^2 - 14 - 174 C}{f^{13}} \right) R^{12} \right. \\ + \left( -\frac{95}{18432} \frac{C^2 d}{f^{12}} - \frac{5}{18432} \frac{-3 - 25 C + 19 C^2}{f^{11}} \right) R^{10} \\ + \left( \frac{9}{1024} \frac{C^2 d}{f^{10}} + \frac{1}{5120} \frac{45 C^2 - 48 C - 10}{f^9} \right) R^8 + \left( -\frac{3}{320} \frac{C^2 d}{f^8} - \frac{1}{1280} \frac{-4 - 11 C + 12 C^2}{f^7} \right) R^6 \Bigg) \\ \Delta^2 + \left( \left( \frac{21}{4096} \frac{C^2 d}{f^{13}} + \frac{1}{12288} \frac{63 C^2 - 2 - 93 C}{f^{12}} \right) R^{12} \right. \\ + \left( -\frac{205}{18432} \frac{C^2 d}{f^{11}} - \frac{5}{18432} \frac{-2 - 55 C + 41 C^2}{f^{10}} \right) R^{10} \\ + \left( \frac{5}{256} \frac{C^2 d}{f^9} + \frac{1}{1280} \frac{25 C^2 - 28 C - 2}{f^8} \right) R^8 + \left( -\frac{7}{320} \frac{C^2 d}{f^7} - \frac{1}{320} \frac{-1 - 7 C + 7 C^2}{f^6} \right) R^6 \Bigg) \Psi \Delta \\ + \left( \left( \frac{133}{49152} \frac{C^2 d}{f^{12}} + \frac{1}{49152} \frac{133 C^2 - 198 C + 8}{f^{11}} \right) R^{12} \right. \\ + \left( -\frac{55}{9216} \frac{C^2 d}{f^{10}} - \frac{5}{36864} \frac{44 C^2 + 3 - 60 C}{f^9} \right) R^{10} \\ + \left( \frac{11}{1024} \frac{C^2 d}{f^8} + \frac{1}{5120} \frac{-64 C + 4 + 55 C^2}{f^7} \right) R^8 + \left( -\frac{1}{80} \frac{C^2 d}{f^6} - \frac{1}{1280} \frac{(16 C - 1)(C - 1)}{f^5} \right) R^6 \Bigg) \\ \Psi^2$$

ZernikeSeries[2812]: calculating the coefficients [6, 2]...

ZernikeSeries[3109]: A[6, 2]:

$$\left( \left( \frac{49}{18432} \frac{C^2 d}{f^{14}} + \frac{7}{147456} \frac{56 C^2 - 6 - 81 C}{f^{13}} \right) R^{12} + \left( -\frac{35}{6144} \frac{C^2 d}{f^{12}} - \frac{5}{6144} \frac{7 C^2 - 1 - 9 C}{f^{11}} \right) R^{10} \right)$$

$$\begin{aligned}
& + \left( \frac{5}{512} \frac{C^2 d}{f^{10}} + \frac{1}{512} \frac{5 C^2 - 5 C - 1}{f^9} \right) R^8 + \left( -\frac{1}{96} \frac{C^2 d}{f^8} - \frac{1}{960} \frac{-3 + 10 C^2 - 8 C}{f^7} \right) R^6 \right) \Delta^2 + \\
& \left( \frac{413}{73728} \frac{C^2 d}{f^{13}} + \frac{7}{73728} \frac{-86 C - 1 + 59 C^2}{f^{12}} \right) R^{12} \\
& + \left( -\frac{25}{2048} \frac{C^2 d}{f^{11}} - \frac{1}{6144} \frac{75 C^2 - 98 C - 2}{f^{10}} \right) R^{10} + \left( \frac{11}{512} \frac{C^2 d}{f^9} + \frac{1}{1024} \frac{-1 - 23 C + 22 C^2}{f^8} \right) R^8 \\
& + \left( -\frac{23}{960} \frac{C^2 d}{f^7} - \frac{1}{960} \frac{23 C^2 - 20 C - 2}{f^6} \right) R^6 \right) \Psi \Delta + \\
& \left( \frac{217}{73728} \frac{C^2 d}{f^{12}} + \frac{7}{147456} \frac{62 C^2 + 4 - 91 C}{f^{11}} \right) R^{12} + \left( -\frac{5}{768} \frac{C^2 d}{f^{10}} - \frac{1}{6144} \frac{40 C^2 + 3 - 53 C}{f^9} \right) R^{10} \\
& + \left( \frac{3}{256} \frac{C^2 d}{f^8} + \frac{1}{1024} \frac{(12 C - 1)(C - 1)}{f^7} \right) R^8 + \left( -\frac{13}{960} \frac{C^2 d}{f^6} - \frac{1}{960} \frac{1 + 13 C^2 - 12 C}{f^5} \right) R^6 \right) \\
& \Psi^2
\end{aligned}$$

ZernikeSeries[3109]: calculating the coefficients [6, 4]...

ZernikeSeries[3327]: calculating the coefficients [6, 6]...

ZernikeSeries[3812]: calculating the coefficients [7, 1]...

ZernikeSeries[3933]: A[7, 1]:

$$\begin{aligned}
& \left( \left( \frac{1}{43008} \frac{C^2 (-1106 + 1259 C) d}{f^{14}} + \frac{1}{43008} \frac{1259 C^4 - 3212 C^3 - 7 C + 70 + 1611 C^2}{f^{13} C} \right) R^{11} \right. \\
& + \left. \left( -\frac{1}{8064} \frac{C^2 (-295 + 286 C) d}{f^{12}} - \frac{1}{16128} \frac{770 C^2 - 1483 C^3 + 572 C^4 - 25 C + 70}{f^{11} C} \right) R^9 \right. \\
& + \left. \left( \frac{1}{4480} \frac{C^2 (-140 + 111 C) d}{f^{10}} + \frac{1}{4480} \frac{146 C + 15 + 111 C^3 - 286 C^2}{f^9} \right) R^7 \right) \Delta^3 + \\
& \left( \frac{1}{7168} \frac{C^2 (743 C - 475) d}{f^{13}} + \frac{1}{43008} \frac{-10342 C^3 + 4458 C^4 + 4290 C^2 + 64 C - 65}{f^{12} C} \right) R^{11} \\
& + \left. \left( -\frac{1}{8064} \frac{C^2 (1063 C - 825) d}{f^{11}} - \frac{1}{16128} \frac{2244 C^2 - 4992 C^3 + 72 C + 2126 C^4 - 65}{f^{10} C} \right) R^9 \right. \\
& + \left. \left( \frac{9}{4480} \frac{C^2 (49 C - 48) d}{f^9} + \frac{3}{4480} \frac{2 - 342 C^2 + 147 C^3 + 164 C}{f^8} \right) R^7 \right) \Psi \Delta^2 + \left( \left( \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{5}{43008} \frac{C^2 (-420 + 1039 C) d}{f^{12}} + \frac{1}{43008} \frac{-12 C - 65 - 10868 C^3 + 5195 C^4 + 3329 C^2}{f^{11} C} \right) R^{11} \\
& + \left( -\frac{1}{8064} \frac{C^2 (1291 C - 683) d}{f^{10}} - \frac{1}{16128} \frac{-5453 C^3 + 11 C - 65 + 1962 C^2 + 2582 C^4}{f^9 C} \right) R^9 \\
& + \left( \frac{1}{4480} \frac{C^2 (567 C - 404) d}{f^8} + \frac{1}{4480} \frac{567 C^3 - 17 + 507 C - 1185 C^2}{f^7} \right) R^7 \Bigg) \Psi^2 - \frac{1}{10752} \frac{R^{11}}{f^{11}} \\
& + \frac{1}{4032} \frac{R^9}{f^9} - \frac{1}{2240} \frac{R^7}{f^7} \Bigg) \Delta + \Bigg( \\
& \left( \frac{1}{10752} \frac{C^2 (-86 + 499 C) d}{f^{11}} + \frac{1}{129024} \frac{-11178 C^3 + 17 + 1890 C^2 + 5988 C^4}{f^{10} C} \right) R^{11} \\
& + \left( -\frac{1}{4032} \frac{C^2 (-74 + 257 C) d}{f^9} - \frac{1}{48384} \frac{3084 C^4 + 1416 C^2 - 5802 C^3 + 17 - 21 C}{f^8 C} \right) R^9 \\
& + \left( \frac{3}{4480} \frac{C^2 (-36 + 79 C) d}{f^7} + \frac{1}{13440} \frac{711 C^3 - 8 - 1323 C^2 + 465 C}{f^6} \right) R^7 \Bigg) \Psi^3 \\
& + \left( -\frac{1}{10752} \frac{R^{11}}{f^{10}} + \frac{1}{4032} \frac{R^9}{f^8} - \frac{1}{2240} \frac{R^7}{f^6} \right) \Psi
\end{aligned}$$

ZernikeSeries[3933]: calculating the coefficients [7, 3]...

ZernikeSeries[4052]: A[7, 3]:

$$\begin{aligned}
& \left( \left( \frac{1}{30720} \frac{C^2 (-280 + 359 C) d}{f^{14}} + \frac{1}{30720} \frac{-878 C^3 + C + 359 C^4 + 14 + 405 C^2}{f^{13} C} \right) R^{11} \right. \\
& + \left. \left( -\frac{1}{768} \frac{C^2 (-10 + 11 C) d}{f^{12}} - \frac{1}{5376} \frac{77 C^4 + 7 - 2 C - 189 C^3 + 90 C^2}{f^{11} C} \right) R^9 \right. \\
& + \left. \left( \frac{1}{896} \frac{C^2 (9 C - 10) d}{f^{10}} + \frac{1}{2688} \frac{30 C + 3 - 64 C^2 + 27 C^3}{f^9} \right) R^7 \right) \Delta^3 + \Bigg( \\
& \left( \frac{1}{7680} \frac{C^2 (313 C - 165) d}{f^{13}} + \frac{1}{30720} \frac{1252 C^4 + 996 C^2 - 2754 C^3 - 12 + 7 C}{f^{12} C} \right) R^{11} \\
& + \left( -\frac{1}{5376} \frac{C^2 (-180 + 281 C) d}{f^{11}} - \frac{1}{10752} \frac{-12 + 488 C^2 + 10 C - 1233 C^3 + 562 C^4}{f^{10} C} \right) R^9
\end{aligned}$$

$$\begin{aligned}
& + \left( \frac{1}{2688} \frac{C^2 (-86+105 C) d}{f^9} + \frac{1}{2688} \frac{C (-221 C+105 C^2+97)}{f^8} \right) R^7 \right) \Psi \Delta^2 + \left( \right. \\
& \left. \left( \frac{1}{10240} \frac{C^2 (-150+479 C) d}{f^{12}} + \frac{1}{30720} \frac{-2862 C^3+2 C-21+731 C^2+1437 C^4}{f^{11} C} \right) R^{11} \right. \\
& \left. + \left( -\frac{1}{5376} \frac{C^2 (-142+335 C) d}{f^{10}} - \frac{1}{10752} \frac{414 C^2+670 C^4+8 C-21-1330 C^3}{f^9 C} \right) R^9 \right. \\
& \left. + \left( \frac{1}{2688} \frac{C^2 (-79+132 C) d}{f^8} + \frac{1}{2688} \frac{(2 C-1)(66 C^2-93 C+4)}{f^7} \right) R^7 \right) \Psi^2 \Delta + \left( \right. \\
& \left. \left( \frac{1}{15360} \frac{C^2 (272 C-35) d}{f^{11}} + \frac{1}{30720} \frac{544 C^3+140 C-986 C^2+1}{f^{10}} \right) R^{11} \right. \\
& \left. + \left( -\frac{1}{5376} \frac{C^2 (131 C-32) d}{f^9} - \frac{1}{10752} \frac{106 C-475 C^2+262 C^3-1}{f^8} \right) R^9 \right. \\
& \left. \left. + \left( \frac{1}{2688} \frac{C^2 (54 C-23) d}{f^7} + \frac{1}{2688} \frac{-95 C^2+34 C+54 C^3-1}{f^6} \right) R^7 \right) \Psi^3 \right)
\end{aligned}$$

ZernikeSeries[4052]: calculating the coefficients [7, 5]...

ZernikeSeries[4163]: calculating the coefficients [7, 7]...

ZernikeSeries[4458]: calculating the coefficients [8, 0]...

ZernikeSeries[4547]: A[8, 0]:

$$\begin{aligned}
& \left( \left( \frac{153}{180224} \frac{C^2 d}{f^{14}} + \frac{9}{1261568} \frac{119 C^2-14-174 C}{f^{13}} \right) R^{12} \right. \\
& \left. + \left( -\frac{19}{14336} \frac{C^2 d}{f^{12}} - \frac{1}{14336} \frac{-3-25 C+19 C^2}{f^{11}} \right) R^{10} \right. \\
& \left. + \left( \frac{9}{7168} \frac{C^2 d}{f^{10}} + \frac{1}{35840} \frac{45 C^2-48 C-10}{f^9} \right) R^8 \right) \Delta^2 + \left( \right. \\
& \left. \left( \frac{81}{45056} \frac{C^2 d}{f^{13}} + \frac{9}{315392} \frac{63 C^2-2-93 C}{f^{12}} \right) R^{12} \right. \\
& \left. + \left( -\frac{41}{14336} \frac{C^2 d}{f^{11}} - \frac{1}{14336} \frac{-2-55 C+41 C^2}{f^{10}} \right) R^{10} \right. \\
& \left. \left. + \left( \frac{5}{1792} \frac{C^2 d}{f^9} + \frac{1}{8960} \frac{25 C^2-28 C-2}{f^8} \right) R^8 \right) \Psi \Delta + \left( \right. \right)
\end{aligned}$$

$$\left( \frac{171}{180224} \frac{C^2 d}{f^{12}} + \frac{9}{1261568} \frac{133 C^2 - 198 C + 8}{f^{11}} \right) R^{12} \\ + \left( -\frac{11}{7168} \frac{C^2 d}{f^{10}} - \frac{1}{28672} \frac{44 C^2 + 3 - 60 C}{f^9} \right) R^{10} \\ + \left( \frac{11}{7168} \frac{C^2 d}{f^8} + \frac{1}{35840} \frac{-64 C + 4 + 55 C^2}{f^7} \right) R^8 \right) \Psi^2$$

ZernikeSeries[4547]: calculating the coefficients [8, 2]...

ZernikeSeries[4764]: A[8, 2]:

$$\left( \left( \frac{21}{22528} \frac{C^2 d}{f^{14}} + \frac{3}{180224} \frac{56 C^2 - 6 - 81 C}{f^{13}} \right) R^{12} \right. \\ + \left( -\frac{3}{2048} \frac{C^2 d}{f^{12}} - \frac{3}{14336} \frac{7 C^2 - 1 - 9 C}{f^{11}} \right) R^{10} + \left( \frac{5}{3584} \frac{C^2 d}{f^{10}} + \frac{1}{3584} \frac{5 C^2 - 5 C - 1}{f^9} \right) R^8 \Big) \Delta^2 \\ + \left( \left( \frac{177}{90112} \frac{C^2 d}{f^{13}} + \frac{3}{90112} \frac{-86 C - 1 + 59 C^2}{f^{12}} \right) R^{12} \right. \\ + \left( -\frac{45}{14336} \frac{C^2 d}{f^{11}} - \frac{3}{71680} \frac{75 C^2 - 98 C - 2}{f^{10}} \right) R^{10} \\ + \left. \left( \frac{11}{3584} \frac{C^2 d}{f^9} + \frac{1}{7168} \frac{-1 - 23 C + 22 C^2}{f^8} \right) R^8 \right) \Psi \Delta + \left( \right. \\ \left( \frac{93}{90112} \frac{C^2 d}{f^{12}} + \frac{3}{180224} \frac{62 C^2 + 4 - 91 C}{f^{11}} \right) R^{12} \\ + \left( -\frac{3}{1792} \frac{C^2 d}{f^{10}} - \frac{3}{71680} \frac{40 C^2 + 3 - 53 C}{f^9} \right) R^{10} \\ + \left. \left( \frac{3}{1792} \frac{C^2 d}{f^8} + \frac{1}{7168} \frac{(12 C - 1)(C - 1)}{f^7} \right) R^8 \right) \Psi^2$$

ZernikeSeries[4764]: calculating the coefficients [8, 4]...

ZernikeSeries[5007]: calculating the coefficients [8, 6]...

ZernikeSeries[5144]: calculating the coefficients [8, 8]...

ZernikeSeries[5534]: calculating the coefficients [9, 1]...

ZernikeSeries[5725]: A[9, 1]:

$$\left( \left( \frac{5}{946176} \frac{C^2 (-1106 + 1259 C) d}{f^{14}} + \frac{5}{946176} \frac{1259 C^4 - 3212 C^3 - 7 C + 70 + 1611 C^2}{f^{13} C} \right) R^{11} \right.$$

$$\begin{aligned}
& + \left( -\frac{1}{64512} \frac{C^2 (-295 + 286 C) d}{f^{12}} - \frac{1}{129024} \frac{770 C^2 - 1483 C^3 + 572 C^4 - 25 C + 70}{f^{11} C} \right) R^9 \Delta^3 \\
& + \left( \begin{aligned}
& \left( \frac{5}{157696} \frac{C^2 (743 C - 475) d}{f^{13}} + \frac{5}{946176} \frac{-10342 C^3 + 4458 C^4 + 4290 C^2 + 64 C - 65}{f^{12} C} \right) R^{11} \\
& + \left( -\frac{1}{64512} \frac{C^2 (1063 C - 825) d}{f^{11}} - \frac{1}{129024} \frac{2244 C^2 - 4992 C^3 + 72 C + 2126 C^4 - 65}{f^{10} C} \right) R^9
\end{aligned} \right) \\
& \psi \Delta^2 + \left( \begin{aligned}
& \left( \frac{25}{946176} \frac{C^2 (-420 + 1039 C) d}{f^{12}} + \frac{5}{946176} \frac{-12 C - 65 - 10868 C^3 + 5195 C^4 + 3329 C^2}{f^{11} C} \right) R^{11} \\
& + \left( -\frac{1}{64512} \frac{C^2 (1291 C - 683) d}{f^{10}} - \frac{1}{129024} \frac{-5453 C^3 + 11 C - 65 + 1962 C^2 + 2582 C^4}{f^9 C} \right) R^9
\end{aligned} \right) \\
& \left( \psi^2 - \frac{5}{236544} \frac{R^{11}}{f^{11}} + \frac{1}{32256} \frac{R^9}{f^9} \right) \Delta + \left( \begin{aligned}
& \left( \frac{5}{236544} \frac{C^2 (-86 + 499 C) d}{f^{11}} + \frac{5}{2838528} \frac{-11178 C^3 + 17 + 1890 C^2 + 5988 C^4}{f^{10} C} \right) R^{11} \\
& + \left( -\frac{1}{32256} \frac{C^2 (-74 + 257 C) d}{f^9} - \frac{1}{387072} \frac{3084 C^4 + 1416 C^2 - 5802 C^3 + 17 - 21 C}{f^8 C} \right) R^9
\end{aligned} \right) \\
& \psi^3 + \left( -\frac{5}{236544} \frac{R^{11}}{f^{10}} + \frac{1}{32256} \frac{R^9}{f^8} \right) \psi
\end{aligned}$$

ZernikeSeries[5725]: calculating the coefficients [9, 3]...

ZernikeSeries[5889]: A[9, 3]:

$$\begin{aligned}
& \left( \left( \frac{1}{135168} \frac{C^2 (-280 + 359 C) d}{f^{14}} + \frac{1}{135168} \frac{-878 C^3 + C + 359 C^4 + 14 + 405 C^2}{f^{13} C} \right) R^{11} \right. \\
& \left. + \left( -\frac{1}{6144} \frac{C^2 (-10 + 11 C) d}{f^{12}} - \frac{1}{43008} \frac{77 C^4 + 7 - 2 C - 189 C^3 + 90 C^2}{f^{11} C} \right) R^9 \right) \Delta^3 + \left(
\right.
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{1}{33792} \frac{C^2 (313 C - 165) d}{f^{13}} + \frac{1}{135168} \frac{1252 C^4 + 996 C^2 - 2754 C^3 - 12 + 7 C}{f^{12} C} \right) R^{11} \\
& + \left( -\frac{1}{43008} \frac{C^2 (-180 + 281 C) d}{f^{11}} - \frac{1}{86016} \frac{-12 + 488 C^2 + 10 C - 1233 C^3 + 562 C^4}{f^{10} C} \right) R^9 \Bigg) \Psi \\
& \Delta^2 + \left( \left( \frac{1}{45056} \frac{C^2 (-150 + 479 C) d}{f^{12}} + \frac{1}{135168} \frac{-2862 C^3 + 2 C - 21 + 731 C^2 + 1437 C^4}{f^{11} C} \right) R^{11} \right. \\
& \left. + \left( -\frac{1}{43008} \frac{C^2 (-142 + 335 C) d}{f^{10}} - \frac{1}{86016} \frac{414 C^2 + 670 C^4 + 8 C - 21 - 1330 C^3}{f^9 C} \right) R^9 \right) \Psi^2 \Delta \\
& + \left( \left( \frac{1}{67584} \frac{C^2 (272 C - 35) d}{f^{11}} + \frac{1}{135168} \frac{544 C^3 + 140 C - 986 C^2 + 1}{f^{10}} \right) R^{11} \right. \\
& \left. + \left( -\frac{1}{43008} \frac{C^2 (131 C - 32) d}{f^9} - \frac{1}{86016} \frac{106 C - 475 C^2 + 262 C^3 - 1}{f^8} \right) R^9 \right) \Psi^3
\end{aligned}$$

ZernikeSeries[5889]: calculating the coefficients [9, 5]...

ZernikeSeries[6034]: calculating the coefficients [9, 7]...

ZernikeSeries[6222]: calculating the coefficients [9, 9]...

ZernikeSeries[6574]: calculating the coefficients [10, 0]...

ZernikeSeries[6634]: A[10, 0]:

$$\begin{aligned}
& \left( \left( \frac{17}{98304} \frac{C^2 d}{f^{14}} + \frac{1}{688128} \frac{119 C^2 - 14 - 174 C}{f^{13}} \right) R^{12} \right. \\
& \left. + \left( -\frac{19}{129024} \frac{C^2 d}{f^{12}} - \frac{1}{129024} \frac{-3 - 25 C + 19 C^2}{f^{11}} \right) R^{10} \right) \Delta^2 + \left( \right. \\
& \left( \frac{3}{8192} \frac{C^2 d}{f^{13}} + \frac{1}{172032} \frac{63 C^2 - 2 - 93 C}{f^{12}} \right) R^{12} \\
& \left. + \left( -\frac{41}{129024} \frac{C^2 d}{f^{11}} - \frac{1}{129024} \frac{-2 - 55 C + 41 C^2}{f^{10}} \right) R^{10} \right) \Psi \Delta + \left( \right. \\
& \left( \frac{19}{98304} \frac{C^2 d}{f^{12}} + \frac{1}{688128} \frac{133 C^2 - 198 C + 8}{f^{11}} \right) R^{12} \\
& \left. + \left( -\frac{11}{64512} \frac{C^2 d}{f^{10}} - \frac{1}{258048} \frac{44 C^2 + 3 - 60 C}{f^9} \right) R^{10} \right) \Psi^2
\end{aligned}$$

ZernikeSeries[6634]: calculating the coefficients [10, 2]...

ZernikeSeries[6775]: A[10, 2]:

$$\begin{aligned}
& \left( \frac{7}{36864} \frac{C^2 d}{f^{14}} + \frac{1}{294912} \frac{56 C^2 - 6 - 81 C}{f^{13}} \right) R^{12} + \left( -\frac{1}{6144} \frac{C^2 d}{f^{12}} - \frac{1}{43008} \frac{7 C^2 - 1 - 9 C}{f^{11}} \right) R^{10} \\
& \left. \Delta^2 + \left( \left( \frac{59}{147456} \frac{C^2 d}{f^{13}} + \frac{1}{147456} \frac{-86 C - 1 + 59 C^2}{f^{12}} \right) R^{12} \right. \right. \\
& \left. \left. + \left( -\frac{5}{14336} \frac{C^2 d}{f^{11}} - \frac{1}{215040} \frac{75 C^2 - 98 C - 2}{f^{10}} \right) R^{10} \right) \Psi \Delta + \left( \right. \right. \\
& \left. \left. \left( \frac{31}{147456} \frac{C^2 d}{f^{12}} + \frac{1}{294912} \frac{62 C^2 + 4 - 91 C}{f^{11}} \right) R^{12} \right. \\
& \left. \left. + \left( -\frac{1}{5376} \frac{C^2 d}{f^{10}} - \frac{1}{215040} \frac{40 C^2 + 3 - 53 C}{f^9} \right) R^{10} \right) \Psi^2 \right)
\end{aligned}$$

ZernikeSeries[6775]: calculating the coefficients [10, 4]...

ZernikeSeries[6905]: calculating the coefficients [10, 6]...

ZernikeSeries[7068]: calculating the coefficients [10, 8]...

ZernikeSeries[7222]: calculating the coefficients [10, 10]...

ZernikeSeries[7617]: calculating the coefficients [11, 1]...

ZernikeSeries[7817]: A[11, 1]:

$$\begin{aligned}
& \left( \frac{1}{1892352} \frac{C^2 (-1106 + 1259 C) d}{f^{14}} + \frac{1}{1892352} \frac{1259 C^4 - 3212 C^3 - 7 C + 70 + 1611 C^2}{f^{13} C} \right) R^{11} \\
& \Delta^3 + \left( \frac{1}{315392} \frac{C^2 (743 C - 475) d}{f^{13}} + \frac{1}{1892352} \frac{-10342 C^3 + 4458 C^4 + 4290 C^2 + 64 C - 65}{f^{12} C} \right) \\
& R^{11} \Psi \Delta^2 + \left( \right. \\
& \left. \left( \frac{5}{1892352} \frac{C^2 (-420 + 1039 C) d}{f^{12}} + \frac{1}{1892352} \frac{-12 C - 65 - 10868 C^3 + 5195 C^4 + 3329 C^2}{f^{11} C} \right) \right. \\
& R^{11} \Psi^2 - \frac{1}{473088} \frac{R^{11}}{f^{11}} \Big) \Delta \\
& + \left( \frac{1}{473088} \frac{C^2 (-86 + 499 C) d}{f^{11}} + \frac{1}{5677056} \frac{-11178 C^3 + 17 + 1890 C^2 + 5988 C^4}{f^{10} C} \right) R^{11} \Psi^3
\end{aligned}$$

$$-\frac{1}{473088} \frac{R^{11}}{f^{10}} \psi$$

ZernikeSeries[7818]: calculating the coefficients [11, 3]...

ZernikeSeries[8030]: A[11, 3]:

$$\begin{aligned} & \left( \frac{1}{1351680} \frac{C^2 (-280 + 359 C) d}{f^{14}} + \frac{1}{1351680} \frac{-878 C^3 + C + 359 C^4 + 14 + 405 C^2}{f^{13} C} \right) R^{11} \Delta^3 + \\ & \left( \frac{1}{337920} \frac{C^2 (313 C - 165) d}{f^{13}} + \frac{1}{1351680} \frac{1252 C^4 + 996 C^2 - 2754 C^3 - 12 + 7 C}{f^{12} C} \right) R^{11} \psi \Delta^2 \\ & + \left( \frac{1}{450560} \frac{C^2 (-150 + 479 C) d}{f^{12}} + \frac{1}{1351680} \frac{-2862 C^3 + 2 C - 21 + 731 C^2 + 1437 C^4}{f^{11} C} \right) R^{11} \\ & \psi^2 \Delta + \left( \frac{1}{675840} \frac{C^2 (272 C - 35) d}{f^{11}} + \frac{1}{1351680} \frac{544 C^3 + 140 C - 986 C^2 + 1}{f^{10}} \right) R^{11} \psi^3 \end{aligned}$$

ZernikeSeries[8031]: calculating the coefficients [11, 5]...

ZernikeSeries[8168]: calculating the coefficients [11, 7]...

ZernikeSeries[8311]: calculating the coefficients [11, 9]...

ZernikeSeries[8456]: calculating the coefficients [11, 11]...

ZernikeSeries[8835]: calculating the coefficients [12, 0]...

ZernikeSeries[8906]: A[12, 0]:

$$\begin{aligned} & \left( \frac{17}{1081344} \frac{C^2 d}{f^{14}} + \frac{1}{7569408} \frac{119 C^2 - 14 - 174 C}{f^{13}} \right) R^{12} \Delta^2 \\ & + \left( \frac{3}{90112} \frac{C^2 d}{f^{13}} + \frac{1}{1892352} \frac{63 C^2 - 2 - 93 C}{f^{12}} \right) R^{12} \psi \Delta \\ & + \left( \frac{19}{1081344} \frac{C^2 d}{f^{12}} + \frac{1}{7569408} \frac{133 C^2 - 198 C + 8}{f^{11}} \right) R^{12} \psi^2 \end{aligned}$$

ZernikeSeries[8906]: calculating the coefficients [12, 2]...

ZernikeSeries[9053]: A[12, 2]:

$$\begin{aligned} & \left( \frac{7}{405504} \frac{C^2 d}{f^{14}} + \frac{1}{3244032} \frac{56 C^2 - 6 - 81 C}{f^{13}} \right) R^{12} \Delta^2 \\ & + \left( \frac{59}{1622016} \frac{C^2 d}{f^{13}} + \frac{1}{1622016} \frac{-86 C - 1 + 59 C^2}{f^{12}} \right) R^{12} \psi \Delta \\ & + \left( \frac{31}{1622016} \frac{C^2 d}{f^{12}} + \frac{1}{3244032} \frac{62 C^2 + 4 - 91 C}{f^{11}} \right) R^{12} \psi^2 \end{aligned}$$

```

ZernikeSeries[9053]: calculating the coefficients [12, 4]...
ZernikeSeries[9209]: calculating the coefficients [12, 6]...
ZernikeSeries[9358]: calculating the coefficients [12, 8]...
ZernikeSeries[9509]: calculating the coefficients [12, 10]...
ZernikeSeries[9662]: calculating the coefficients [12, 12]...

> cost(S);

$$1605 \text{ additions} + 11173 \text{ multiplications} + 690 \text{ divisions} + 17 \text{ functions}$$


> cost(OPD);

$$433 \text{ additions} + 3710 \text{ multiplications} + 187 \text{ divisions} + 13 \text{ functions}$$


> save OPD, OPD_avg, OPD, S, c, `d:/optics/AESOP/BeamCompExample.eqs`;

```

## Sanity Check

The series expansion of the wavefront is stored in  $S$ , and the series coefficients are stored in the table,  $c$ . As a quick check, let's make sure  $S$  is identical to the OPD:

```

> factor( expand(S-OPD) );

$$0$$


```

## Elimination of Coma, Due to a Mirror Displacement, by a Compensating Mirror Rotation

We can use the function **ZernikeTerm()** to access the individual components. For example, the coma term is

```

> ZernikeTerm(3,1,c);


$$\begin{aligned} & \left( \left( \frac{5}{57344} \frac{C^2 (-1106 + 1259 C) d}{f^{14}} + \frac{5}{57344} \frac{1259 C^4 - 3212 C^3 - 7 C + 70 + 1611 C^2}{f^{13} C} \right) R^{11} \right. \\ & + \left( -\frac{1}{1344} \frac{C^2 (-295 + 286 C) d}{f^{12}} - \frac{1}{2688} \frac{770 C^2 - 1483 C^3 + 572 C^4 - 25 C + 70}{f^{11} C} \right) R^9 \\ & + \left( \frac{1}{320} \frac{C^2 (-140 + 111 C) d}{f^{10}} + \frac{1}{320} \frac{146 C + 15 + 111 C^3 - 286 C^2}{f^9} \right) R^7 \\ & + \left( -\frac{7}{80} \frac{C^2 (-8 + 5 C) d}{f^8} - \frac{1}{80} \frac{35 C^3 + 49 C - 99 C^2 + 9}{f^7} \right) R^5 \\ & + \left( \frac{1}{24} \frac{C^2 (-17 + 8 C) d}{f^6} + \frac{1}{48} \frac{(4 C + 1)(4 C^2 - 14 C + 9)}{f^5} \right) R^3 \Bigg) \Delta^3 + \left( \right. \\ & \left. \frac{15}{28672} \frac{C^2 (743 C - 475) d}{f^{13}} + \frac{5}{57344} \frac{-10342 C^3 + 4458 C^4 + 4290 C^2 + 64 C - 65}{f^{12} C} \right) R^{11} \end{aligned}$$


```

$$\begin{aligned}
& + \left( -\frac{1}{1344} \frac{C^2 (1063 C - 825) d}{f^{11}} - \frac{1}{2688} \frac{2244 C^2 - 4992 C^3 + 72 C + 2126 C^4 - 65}{f^{10} C} \right) R^9 \\
& + \left( \frac{9}{320} \frac{C^2 (49 C - 48) d}{f^9} + \frac{3}{320} \frac{2 - 342 C^2 + 147 C^3 + 164 C}{f^8} \right) R^7 \\
& + \left( -\frac{3}{80} \frac{C^2 (51 C - 65) d}{f^7} - \frac{1}{80} \frac{7 + 191 C + 153 C^3 - 388 C^2}{f^6} \right) R^5 \\
& + \left( \frac{1}{24} \frac{C^2 (40 C - 69) d}{f^5} + \frac{1}{48} \frac{80 C^3 - 230 C^2 + 104 C + 13}{f^4} \right) R^3 \Bigg) \Psi \Delta^2 + \left( \left( \right. \right. \\
& \left. \left. \frac{25}{57344} \frac{C^2 (-420 + 1039 C) d}{f^{12}} + \frac{5}{57344} \frac{-12 C - 65 - 10868 C^3 + 5195 C^4 + 3329 C^2}{f^{11} C} \right) R^{11} \right. \\
& + \left( -\frac{1}{1344} \frac{C^2 (1291 C - 683) d}{f^{10}} - \frac{1}{2688} \frac{-5453 C^3 + 11 C - 65 + 1962 C^2 + 2582 C^4}{f^9 C} \right) R^9 \\
& + \left( \frac{1}{320} \frac{C^2 (567 C - 404) d}{f^8} + \frac{1}{320} \frac{567 C^3 - 17 + 507 C - 1185 C^2}{f^7} \right) R^7 \\
& + \left( -\frac{1}{80} \frac{C^2 (-209 + 214 C) d}{f^6} - \frac{1}{160} \frac{460 C + 428 C^3 - 17 - 974 C^2}{f^5} \right) R^5 \\
& + \left( \frac{1}{3} \frac{C^2 (8 C - 11) d}{f^4} + \frac{1}{48} \frac{-3 + 128 C^3 - 328 C^2 + 156 C}{f^3} \right) R^3 \Bigg) \Psi^2 - \frac{5}{14336} \frac{R^{11}}{f^{11}} + \frac{1}{672} \frac{R^9}{f^9} \\
& - \frac{1}{160} \frac{R^7}{f^7} + \frac{1}{40} \frac{R^5}{f^5} - \frac{1}{12} \frac{R^3}{f^3} \Bigg) \Delta + \left( \right. \\
& \left. \left( \frac{5}{14336} \frac{C^2 (-86 + 499 C) d}{f^{11}} + \frac{5}{172032} \frac{-11178 C^3 + 17 + 1890 C^2 + 5988 C^4}{f^{10} C} \right) R^{11} \right. \\
& + \left( -\frac{1}{672} \frac{C^2 (-74 + 257 C) d}{f^9} - \frac{1}{8064} \frac{3084 C^4 + 1416 C^2 - 5802 C^3 + 17 - 21 C}{f^8 C} \right) R^9 \\
& + \left( \frac{3}{320} \frac{C^2 (-36 + 79 C) d}{f^7} + \frac{1}{960} \frac{711 C^3 - 8 - 1323 C^2 + 465 C}{f^6} \right) R^7 \\
& + \left. \left( -\frac{1}{80} \frac{C^2 (96 C - 67) d}{f^5} - \frac{1}{480} \frac{-19 + 504 C - 1170 C^2 + 576 C^3}{f^4} \right) R^5 \right)
\end{aligned}$$

$$\begin{aligned}
& + \left( \frac{1}{12} \frac{C^2 (-17 + 16 C) d}{f^3} + \frac{1}{144} \frac{-444 C^2 + 216 C + 192 C^3 - 7}{f^2} \right) R^3 \Bigg) \Psi^3 \\
& + \left( -\frac{5}{14336} \frac{R^{11}}{f^{10}} + \frac{1}{672} \frac{R^9}{f^8} - \frac{1}{160} \frac{R^7}{f^6} + \frac{1}{40} \frac{R^5}{f^4} - \frac{1}{12} \frac{R^3}{f^2} \right) \Psi \Bigg) (3 p^3 - 2 p) \cos(\phi)
\end{aligned}$$

Notice that it is linear in both  $\Delta$  and  $\psi$ . Let's see what angle compensates for a translation, to first order in angle.

```
> collect(expansion(ZernikeTerm(3,1,c), psi, 1, ON), [psi,Delta], factor);
```

$$\begin{aligned}
& \left( \frac{1}{860160} R^3 p \cos(\phi) (-2 + 3 p^2) (1863680 f^9 C^2 + 16128 R^4 f^5 C - 680320 R^6 f^3 C^4 \right. \\
& + 1597440 R^6 f^3 C^3 - 23040 R^6 f^3 C - 213750 R^8 C^3 d - 2053632 R^2 f^7 C^2 \\
& + 1322496 R^4 f^5 C^2 + 1185408 R^4 f^5 C^4 - 2757888 R^4 f^5 C^3 - 75264 R^2 f^7 C \\
& + 334350 R^8 f C^4 + 4800 R^8 f C - 775650 R^8 f C^3 + 334350 R^8 C^4 d - 718080 R^6 f^3 C^2 \\
& - 2472960 f^8 C^3 d + 1433600 f^8 C^4 d + 4171776 R^2 f^7 C^3 - 1645056 R^2 f^7 C^4 \\
& + 321750 R^8 f C^2 + 1433600 f^9 C^4 + 2096640 R^2 f^6 C^3 d - 4121600 f^9 C^3 + 232960 f^9 C \\
& - 4875 R^8 f + 1185408 R^4 f^4 C^4 d - 1645056 R^2 f^6 C^4 d - 1161216 R^4 f^4 C^3 d \\
& \left. + 528000 R^6 f^2 C^3 d - 680320 R^6 f^2 C^4 d + 20800 R^6 f^3) \Delta^2 \right/ (f^{13} C) - \frac{1}{215040} \\
& \frac{R^3 p \cos(\phi) (-2 + 3 p^2) (-320 R^6 f^2 + 75 R^8 - 5376 R^2 f^6 + 17920 f^8 + 1344 R^4 f^4)}{f^{10}} \Bigg) \Psi + \\
& \frac{1}{860160} R^3 p \cos(\phi) (-2 + 3 p^2) (394240 f^9 C^2 + 40320 R^4 f^5 C - 183040 R^6 f^3 C^4 \\
& + 474560 R^6 f^3 C^3 + 8000 R^6 f^3 C - 82950 R^8 C^3 d - 526848 R^2 f^7 C^2 + 392448 R^4 f^5 C^2 \\
& + 298368 R^4 f^5 C^4 - 768768 R^4 f^5 C^3 - 96768 R^2 f^7 C + 94425 R^8 f C^4 - 525 R^8 f C \\
& - 240900 R^8 f C^3 + 94425 R^8 C^4 d - 246400 R^6 f^3 C^2 - 609280 f^8 C^3 d + 286720 f^8 C^4 d \\
& + 1064448 R^2 f^7 C^3 - 376320 R^2 f^7 C^4 + 120825 R^8 f C^2 + 286720 f^9 C^4 \\
& + 602112 R^2 f^6 C^3 d - 931840 f^9 C^3 + 161280 f^9 C + 5250 R^8 f + 298368 R^4 f^4 C^4 d \\
& - 376320 R^2 f^6 C^4 d - 376320 R^4 f^4 C^3 d + 188800 R^6 f^2 C^3 d - 183040 R^6 f^2 C^4 d \\
& - 22400 R^6 f^3) \Delta^3 \right/ (f^{14} C) - \frac{1}{215040}
\end{aligned}$$

```


$$\frac{R^3 \rho \cos(\phi) (-2 + 3 \rho^2) (-320 R^6 f^2 + 75 R^8 - 5376 R^2 f^6 + 17920 f^8 + 1344 R^4 f^4) \Delta}{f^{11}}$$

> psi = collect( solve("",psi), [Delta,C], factor );

$$\Psi = -\Delta (((94425 R^8 + 286720 f^8 + 298368 R^4 f^4 - 183040 R^6 f^2 - 376320 R^2 f^6) (f+d) C^4 + (474560 R^6 f^3 - 768768 R^4 f^5 - 82950 R^8 d - 376320 R^4 f^4 d - 609280 f^8 d - 240900 R^8 f + 602112 R^2 f^6 d - 931840 f^9 + 188800 R^6 f^2 d + 1064448 R^2 f^7) C^3 + f(-246400 R^6 f^2 + 392448 R^4 f^4 + 394240 f^8 - 526848 R^2 f^6 + 120825 R^8) C^2 - f(96768 R^2 f^6 - 161280 f^8 - 8000 R^6 f^2 - 40320 R^4 f^4 + 525 R^8) C + 350 R^6 f(15 R^2 - 64 f^2)) \Delta^2 - 4 f^3 (-320 R^6 f^2 + 75 R^8 - 5376 R^2 f^6 + 17920 f^8 + 1344 R^4 f^4) C) / (f((2 (167175 R^8 + 716800 f^8 + 592704 R^4 f^4 - 340160 R^6 f^2 - 822528 R^2 f^6) (f+d) C^4 + (1597440 R^6 f^3 - 2757888 R^4 f^5 - 213750 R^8 d - 1161216 R^4 f^4 d - 2472960 f^8 d - 775650 R^8 f + 2096640 R^2 f^6 d - 4121600 f^9 + 528000 R^6 f^2 d + 4171776 R^2 f^7) C^3 + 2 f(-359040 R^6 f^2 + 661248 R^4 f^4 + 931840 f^8 - 1026816 R^2 f^6 + 160875 R^8) C^2 + 64 f(-1176 R^2 f^6 + 3640 f^8 - 360 R^6 f^2 + 252 R^4 f^4 + 75 R^8) C - 325 R^6 f(15 R^2 - 64 f^2)) \Delta^2 - 4 f^3 (-320 R^6 f^2 + 75 R^8 - 5376 R^2 f^6 + 17920 f^8 + 1344 R^4 f^4) C))
> psi = expansion( rhs(""), Delta, 3, ON );

$$\Psi = -\frac{1}{4} (1469440 f^9 C^2 - 24192 R^4 f^5 C - 497280 R^6 f^3 C^4 + 1122880 R^6 f^3 C^3 - 31040 R^6 f^3 C - 130800 R^8 C^3 d - 1526784 R^2 f^7 C^2 + 930048 R^4 f^5 C^2 + 887040 R^4 f^5 C^4 - 1989120 R^4 f^5 C^3 + 21504 R^2 f^7 C + 239925 R^8 f C^4 + 5325 R^8 f C - 534750 R^8 f C^3 + 239925 R^8 C^4 d - 471680 R^6 f^3 C^2 - 1863680 f^8 C^3 d + 1146880 f^8 C^4 d + 3107328 R^2 f^7 C^3 - 1268736 R^2 f^7 C^4 + 200925 R^8 f C^2 + 1146880 f^9 C^4 + 1494528 R^2 f^6 C^3 d - 3189760 f^9 C^3 + 71680 f^9 C - 10125 R^8 f + 887040 R^4 f^4 C^4 d - 1268736 R^2 f^6 C^4 d - 784896 R^4 f^4 C^3 d + 339200 R^6 f^2 C^3 d - 497280 R^6 f^2 C^4 d + 43200 R^6 f^3) \Delta^3 / (f^4 (-320 R^6 f^2 + 75 R^8 - 5376 R^2 f^6 + 17920 f^8 + 1344 R^4 f^4) C) - \frac{\Delta}{f}$$

> psi = collect( rhs(""), [Delta,R,d], factor );

$$\Psi = -\frac{1}{4} ((75 C^3 (3199 C - 1744) d + 75 f (71 C + 2679 C^2 + 3199 C^4 - 135 - 7130 C^3)) R^8$$$$

```

$$\begin{aligned}
& + (-640 C^3 f^2 (-530 + 777 C) d - 320 f^3 (-3509 C^3 + 1474 C^2 + 97 C - 135 + 1554 C^4)) R^6 \\
& + (5376 C^3 f^4 (-146 + 165 C) d + 2688 f^5 C (-740 C^2 + 346 C - 9 + 330 C^3)) R^4 \\
& + (-10752 C^3 f^6 (-139 + 118 C) d - 10752 f^7 C (118 C^3 - 289 C^2 + 142 C - 2)) R^2 \\
& + 143360 C^3 f^8 (-13 + 8 C) d + 35840 f^9 C (41 C - 89 C^2 + 2 + 32 C^3)) \Delta^3 / (f^4 \\
& (-320 R^6 f^2 + 75 R^8 - 5376 R^2 f^6 + 17920 f^8 + 1344 R^4 f^4) C) - \frac{\Delta}{f}
\end{aligned}$$

For example, with a displacement of 10 microns, a primary focal length of 100 cm, and an input beam radius of 10 cm, the angle that to first order corrects for coma is

```
> epsilon := evalf( subs( Delta=10*10^(-4), R=10,
                           f=100, d=20, C=10, rhs(") )/(1*arcsec) );
epsilon := -2.062651014
```

That is, a bit over two arc seconds, or 10 microradians.

## Analysis of the Wavefront Aberrations

```
> read`d:/optics/AESOP/BeamCompExample.eqs`:
Read the plotting routines and tell them to not chatter:
> read`c:/maple/plotting.p`:
plotting: [jpgplot( plotcommand, filename, [height,width] ),
curveplot2D( curve, t=t1..t2, plot_options ),
implicitplot2D( eqn, x=x1..x2, y=y1..y2, plot_options ),
scalarplot( curve, t=t1..t2, plot_options ),
curveplot3D( curve, t=t1..t2, plot3d_options ),
tubecurve( curve, t, radius, theta ),
tubecurveplot( curve, radius, t=t1..t2, theta=theta1..theta2, plot3d_options ),
implicitplot3D( eqn, x=x1..x2, y=y1..y2, z=z1..z2, plot3d_options )
PlotZernikeWavefront( opd, Zcoeffs, sublist, units, pert, f, R, verbose, plot3d_options )]
```

```
> verbosity := 0:
```

Let's use a beam compression of 10 and a detector distance of 20 cm.

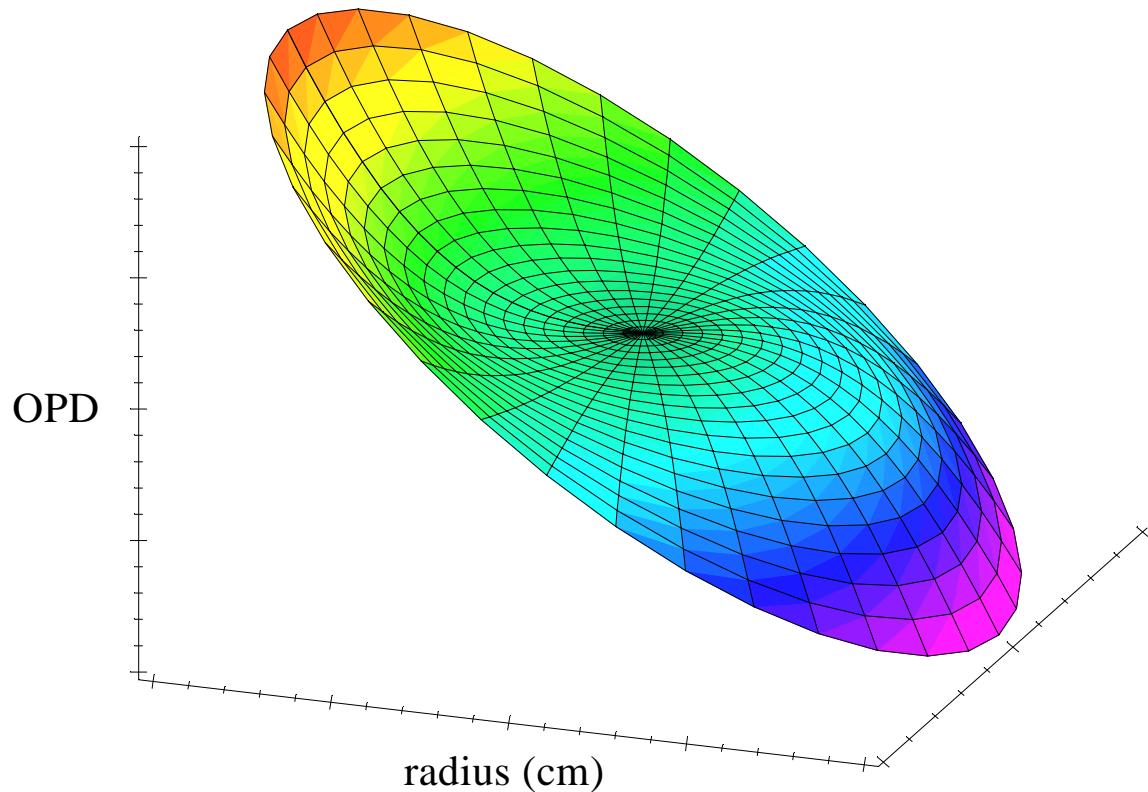
```
> for k from 1 to nops([indices(c)]) do
  c[k][A] := subs( d=20, C=10, c[k][A] );
  c[k][B] := subs( d=20, C=10, c[k][B] );
od:
```

```
|> opd := subs( d=20, C=10, opd );
```

Now let's look at what happens with a 10 micron translation and no rotation. We'll remove the offset and tilt terms first.

```
|> PlotZernikeWavefront( opd, [Delta,psi], c, [[0,0],[1,1]],  
|`microns`, [10*10^(-4),0*arcsec], 100.0, 10.0 );
```

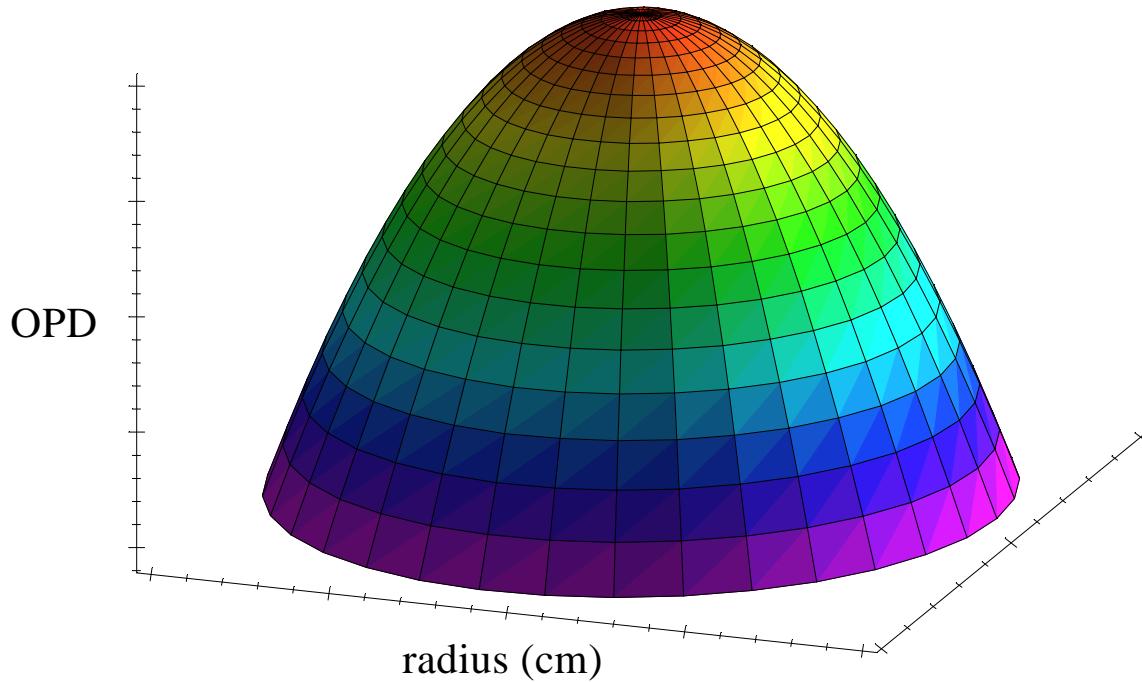
## Wavefront Error (microns)



We're suffering from about two microns of coma at the edge of the beam. Let's now rotate the primary by the amount we calculated above that compensates for the translation.

```
|> PlotZernikeWavefront( opd, [Delta,psi], c, [[0,0],[1,1]],  
|`pm`, [10*10^(-4),epsilon*arcsec], 100.0, 10.0 );
```

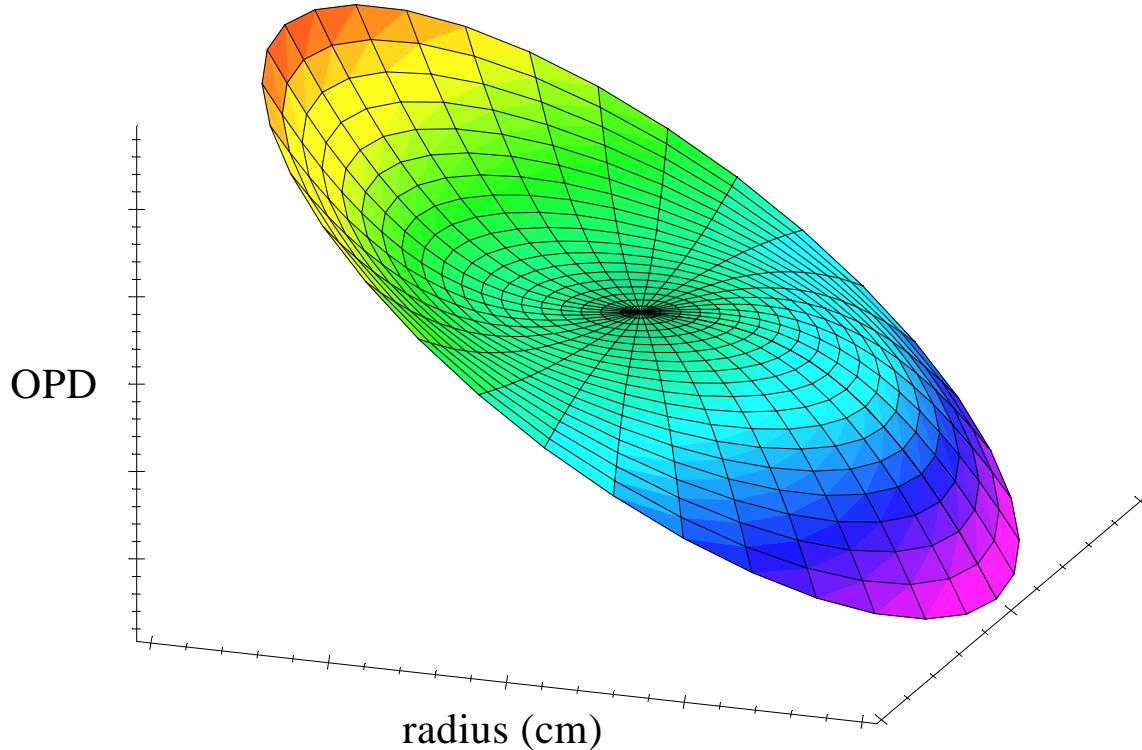
## Wavefront Error (pm)



The coma due to the translation has disappeared, leaving only  $\sim 200$  picometers of defocus. This tells us two things: (1) there is a rotation of the primary mirror that compensate for a lateral displacement, at least as far as the dominant coma aberration is concerned; and (2) coma is by far the dominant aberration. If we remove the series of defocus terms, we have

```
> PlotZernikeWavefront( opd, [Delta,psi], c,
  [[0,0],[1,1],[2,0],[4,0],[6,0],[8,0],[10,0],[12,0]],
  `pm`, [10*10^(-4),epsilon*arcsec], 100.0, 10.0,
  orientation=[-70,65] );
```

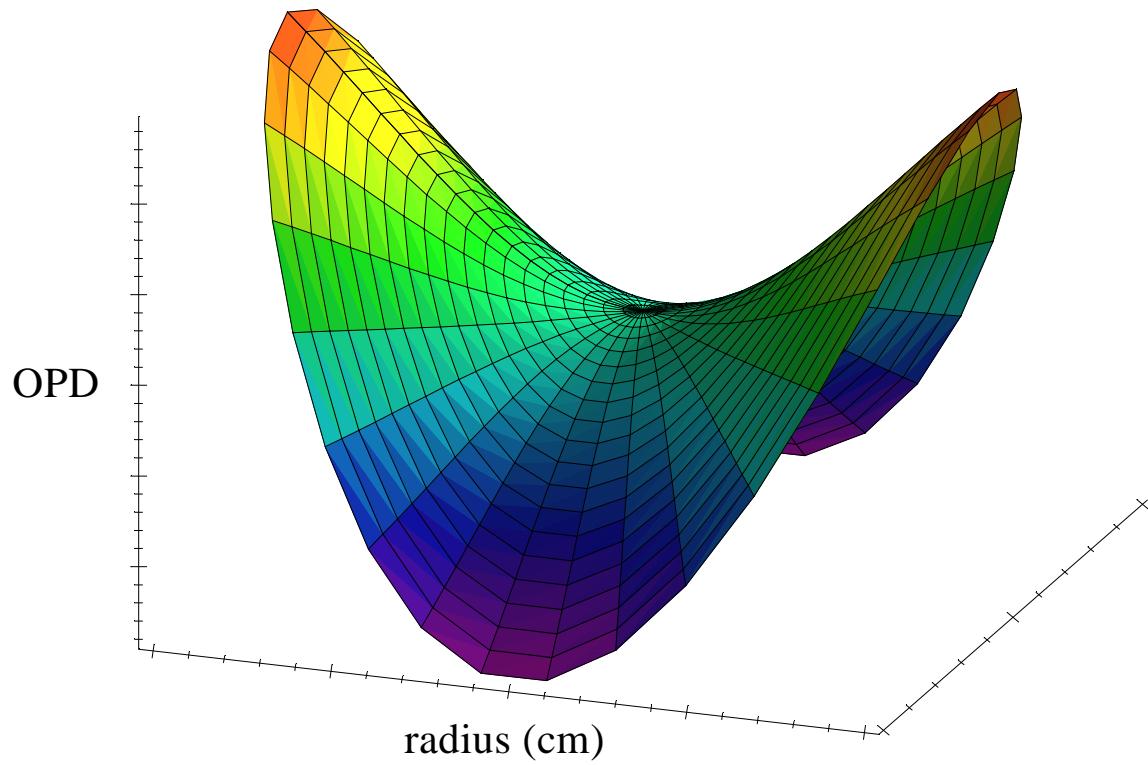
## Wavefront Error (pm)



Now the residual coma terms are apparent at about 3 picometers. We see that the compensating rotation of the primary has reduced the coma aberration by 6 orders of magnitude. The residual is due to the  $\psi^2$  component of the coma, which we did not correct for in the rotation. Let's see what else besides coma that we have left. We should expect a very small astigmatism.

```
> PlotZernikeWavefront( opd, [Delta,psi], c,
  [[0,0],[1,1],[2,0],[4,0],[6,0],[8,0],[10,0],[12,0],
  [3,1],[5,1],[7,1],[9,1],[11,1]],
  `fm`, [10*10^(-4),epsilon*arcsec], 100.0, 10.0 );
```

## Wavefront Error (fm)



Indeed, we have a ~3 femtometer astigmatism residual.